Information in Fixed-Price Trading

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ABSTRACT

Fixed-price venues, such as dark pools, are commonly thought to hinder the incorporation of information into prices. We show that fixed-price venues can produce information beyond what is contained in prices. In a dynamic model of strategic traders with heterogeneous information, cross-period price impact from a fixed-price session deters informed trading on signals at the fixed price, while inventory-based demand is not deterred. Aggregate quantities help to reveal inventories, which the market-clearing price does not fully separate from the fundamental. This information reduces adverse selection and improves price informativeness in subsequent price-clearing sessions. Anticipating the fixed-price session, traders delay trade, reducing liquidity, but improved inventory inference can outweigh this cost, increasing welfare when information is sufficiently asymmetric and the market is large.

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I. Introduction

Prices aggregate information. This is the fundamental insight of Hayek (1945), who argues that market prices communicate dispersed information, and of Grossman (1976), who formalizes how equilibrium prices reflect investors' heterogeneous private information. It is natural to then think that a trading mechanism without a price of its own cannot reveal any further information. Following this logic, practitioners, regulators, and academics have long worried that trading venues where prices are fixed, such as dark pools[1], detract from the information production that prices provide in lit markets (e.g., Securities and Exchange Commission, 2010, Degryse et al., 2014, Hatheway et al., 2017, Buti et al., 2017). The studies that find dark trading need not harm price discovery rely on sorting: dark pools draw the uninformed away from the exchange, so the lit price becomes more informative because of who leaves it, not because dark volume carries information of its own (Zhu, 2013, Comerton-Forde and Putniņš, 2015). In both views, the fixed-price trading venue itself produces no information. We show that it can, and we study its dynamic implications for exchange trade.

A fixed price rules out learning from the price, but not learning from trade in that type of trading mechanism. By observing the quantity that is allocated in the session, each trader can infer the aggregate quantity demanded. Traders therefore learn the information contained in the quantity that clears at the fixed price. The key mechanism is that, when traders are large, rational, and strategic, trades affect beliefs and therefore future prices. So, while there is no contemporaneous price impact, by construction of trading at a fixed price, there is cross-period (intertemporal) price impact, since traders update their beliefs after observing the aggregate quantity demanded in the fixed-price session.

This cross-period price impact is large. Because there is no contemporaneous price impact, a trader is tempted to deviate by trading more aggressively on their private information than the others expect. The other traders still believe that everyone is playing the equilibrium strategy, so the deviation distorts their beliefs. In equilibrium, the cross-period price impact that would follow such a deviation must be large enough that trading at worse prices in the future costs the trader more than they gain from trading aggressively in the fixed-price session. As a result, the informed trade little on their signals in the fixed-price session. Inventory-driven trade is not deterred in the same way, because traders do not unwind it in

[1]Other examples of fixed-price trading mechanisms are workups in the interdealer Treasury and index CDS markets and matching sessions in corporate bonds and swaps.

the next session, so a price that moves against them costs them little. The aggregate quantity is therefore disproportionately a signal about inventories. The market-clearing price, on the other hand, places relatively more weight on the fundamental value of the asset. Information regarding counterparties' inventories is useful because it reduces adverse selection in future price-clearing sessions. Traders face less price impact in subsequent price-clearing sessions, because large demands are more likely to be attributed to offloading large inventories than to exploiting large private signals.

To formalize this argument, we study a dynamic model of strategic trade in which large traders differ in the quality of their information and a fixed-price session may follow a price-clearing session. We study a model with two types of traders, informed and uninformed, who trade to manage inventories and differ in the precision of their private signals about the value of the asset. Trade takes place over two periods. Traders first trade in a price-clearing session, which may be followed by a fixed-price session that executes at the prior market-clearing price, and then trade again in a second price-clearing session. Dynamic models of imperfect competition typically abstract from heterogeneous information (e.g., Vayanos, 1999, Du and Zhu, 2017, Antill and Duffie, 2020, Chen and Duffie, 2021). With heterogeneous information, beliefs depend on the entire history of prices, and a trader's demand today changes what others infer in every future session, so learning is non-Markovian, and the equilibrium cannot be found by standard Bellman equations (Rostek et al., 2025). The fixed-price session adds to this difficulty. We solve the model by extending the non-recursive approach of Rostek et al. (2025), in which the cross-period price and demand impacts are constants in a linear-quadratic setting and are determined jointly as a fixed point.

The model yields three main results. First, the aggregate quantity submitted in the fixed-price session is informative relative to the prevailing market-clearing price. Given the market-clearing price, the aggregate quantity submitted is almost entirely a signal about inventories. In our baseline parameterization, it reduces the uninformed's uncertainty about the value of the asset by roughly 80%. This reduction in uncertainty reduces adverse selection and price impact in the subsequent price-clearing session. Second, the fixed-price session does introduce some inefficiencies. Traders who anticipate the fixed-price session delay part of their inventory adjustment by shading their demands in the price-clearing session, which lowers liquidity there whether or not the fixed-price session takes place (Antill and Duffie, 2020, Blonien, 2026). Third, the information acquired in the fixed-price session can outweigh

the cost of this delay, making the addition of a fixed-price session welfare improving. The benefits and costs are shared by different traders. The uninformed gain because they learn from the aggregate quantity, while the informed lose part of their trading profits. The fixed-price session raises aggregate welfare when the uninformed's gains outweighs the informed's losses. This requires sufficient asymmetric information for learning from quantities to reduce, and a market large enough that the strategic delay in price-clearing sessions preceding the fixed-price session is small.

Related Literature

Our paper contributes to three strands of the literature. The first studies how trade is affected by the addition of fixed-price trading sessions, sometimes referred to as size discovery sessions. Duffie and Zhu (2017) show that a fixed-price session held before trading begins improves allocative efficiency, and Antill and Duffie (2020) show that when trade is continuous and a fixed-price session may occur in the future, traders delay trade in anticipation, which lowers allocative efficiency. Blonien (2026) shows that the net effect depends on the frequency of trade and the degree of adverse selection. In each of these papers, the fixed-price session reallocates inventory but reveals no information relative to the prevailing market-clearing price. These studies only have a single type of trader who is ex-ante identical, so the price already reveals everything traders can learn from each other. We show that when traders differ in the quality of their information, the aggregate quantity is informative, which gives the fixed-price session a further welfare benefit. Empirically, the quantities traded at a fixed price do contain information. Fleming and Nguyen (2018) find that shocks to order flow in U.S. Treasury workups (fixed-price sessions) explain 6–8% of the variation in returns on benchmark notes and about 10% of the variation in the level of the yield curve. They also find that the information in workup order flow differs from the information in the trades that set the price: information related to dealers' proprietary client order flow is more likely to show up in workup trades, while information from public announcements is more likely to show up in pre-workup trades. This is broadly consistent with the separation of trade into fundamental and non-fundamental components that we find.

The second strand studies the information content that market participants learn from quantities. Blume et al. (1994) and Schneider (2009) show that trading volume carries information that cannot be deduced from the price, because it reveals the precision of traders'

information. Ganguli and Yang (2009) and Manzano and Vives (2011) study markets in which the price reflects a second source of variation, the supply of the asset or traders' exposure to aggregate risk, about which traders are privately informed. In our model, the second source of variation is inventories, and the aggregate quantity is a public signal about them that lets traders separate the two components, fundamental and non-fundamental, of the information contained in the price. This signal arises endogenously from trade at a fixed price, because the cross-period price impact deters the informed from trading on their signals in the fixed-price session. Our mechanism also differs from the literature on the choice between exchanges and fixed-price venues (Hendershott and Mendelson, 2000, Zhu, 2013, Buti et al., 2017, Menkveld et al., 2017). In Zhu (2013), a dark pool improves price discovery because the informed self-select onto the exchange. In our model, the sessions are sequential, every trader participates in both, and the information arises from the quantity submitted at the fixed price.

The third strand studies dynamic trade under imperfect competition. Strategic traders split their orders over time and across venues to reduce price impact, which can reduce liquidity in any one session (Vayanos, 1999, Du and Zhu, 2017, Sannikov and Skrzypacz, 2016, Chen and Duffie, 2021, Blonien and Ober, 2026). With private information, traders also learn from prices, and price impact reflects adverse selection (Kyle, 1989, Vives, 2011, Rostek and Weretka, 2012). Dynamic models with traders who differ in the quality of their information are difficult to solve, because a trade today changes what others infer in every future session. Rostek et al. (2025) develop a non-recursive approach to this problem and show that making a statistic of past trades public introduces dynamic adverse selection, which can raise price impact and lower welfare. We apply their approach to a market in which the public statistic is generated by a fixed-price session, and we show that intertemporal price impact modifies traders' demands in fixed-price sessions, which implies the aggregate quantity is informative.

II. The Model

In this section, we set up a dynamic model of trade among a finite number of rational large traders. Because trading behavior today affects future beliefs, the resulting equilibrium is neither stationary nor Markovian (Rostek et al. (2025)). The way trading behavior affects future beliefs will also differ by trading mechanism. We study a market with two types of traders who both experience inventory shocks, but the informed traders also observe a noisy

private signal of a dividend's liquidation value, while uninformed traders do not. We assume there are three trading sessions across two periods.[2]

Figure 1 shows the setup of the model. There are two periods during which trade occurs, denoted period 1 and period 2. As we will see later, there will be potentially two successive trading opportunities in period 1. Each period, traders exchange a divisible asset that pays a dividend of v_t upon liquidation per unit of the asset. Before trade in the first period, the liquidating value is uncertain and equal to $\tilde{v}_1 \sim N(0, \sigma^2)$.[3] Before the second period of trade, the liquidating value will be shocked to $\tilde{v}_2 \sim N(v_1, \sigma_\eta^2)$.

There are two types of traders. There are informed traders ($N \in \mathbb{Z}_{\geq 0}$) who receive private independent noisy signals about the liquidation value of the asset at a given time, while uninformed traders ($M \in \mathbb{Z}_{\geq 0}$) do not, but will learn about the liquidation value through market outcomes. Before trade each period, the private noisy signal informed traders receive is $S_{i,t} = v_t + \epsilon_{i,t}$, where $\epsilon_{i,t} \overset{\text{iid}}{\sim} N(0, \sigma_\epsilon^2)$. Throughout, we write $\tau \in \{I, U\}$ for a trader's type, with $-\tau$ denoting the opposite type. For trade to occur in the equilibria we consider, we require $N + M > 2$.

In the model, traders dynamically manage inventory positions. Each trader, i, has an inventory position at the beginning of period t of z_t^i. Before each trading session, each trader receives a random endowment shock to their inventory, denoted $h_t^i \sim N(0, \sigma_h^2)$. For simplicity, assume all traders have an initial inventory of zero before the first shock. In each period, traders engage in trade to manage their inventory positions z_t^i. Traders are risk-neutral but face quadratic costs for holding inventory, representing risk management and collateral considerations. Chen and Duffie (2021), Antill and Duffie (2020), Duffie and Zhu (2017), Du and Zhu (2017), Sannikov and Skrzypacz (2016), Rostek and Weretka (2012), Vives (2011), Blonien and Ober (2026), Blonien (2026) and Chen (2022) all use a similar quadratic holding cost. More generally, including these exogenous inventory costs is a reduced-form approach to modeling incentives to risk share. For simplicity, we focus on the case where the marginal holding cost for all traders is governed by the same parameter, $\gamma > 0$.[4]

We describe the trading sessions further below. With probability $1 - \delta$ the asset will liquidate after period 1, and otherwise, with probability δ, the asset will liquidate after

[2]The model can be extended to have more types of traders and additional trading periods, though such extensions greatly complicate computation and obscure the main economic insights.

[3]Adding a non-zero mean has no qualitative effects.

[4]This assumption can also be relaxed, though it is not the focus of our paper.

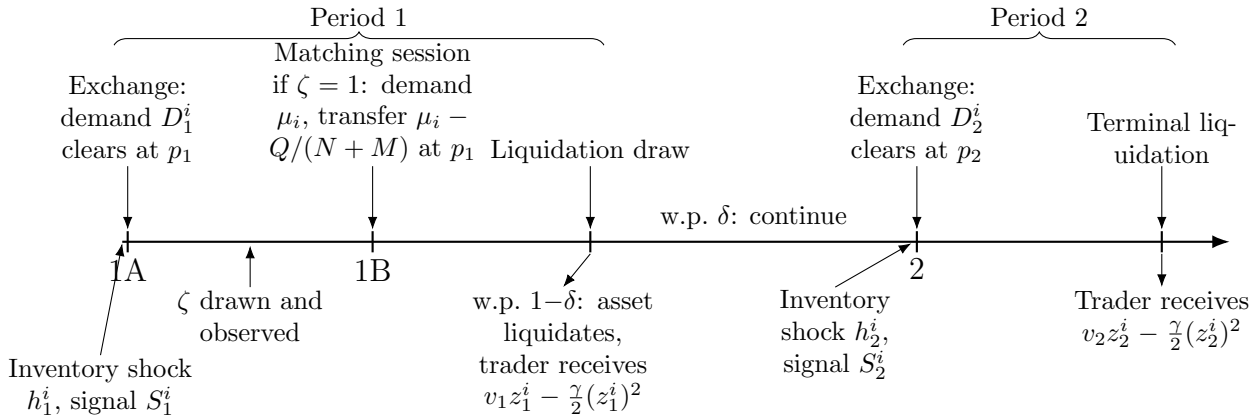

Figure 1: Timeline of the model

The model begins with each trader receiving a random inventory shock and a private signal about the asset's value. Period 1 trade takes place in two stages. In stage 1A, traders submit demand schedules to an exchange that clears at a single price. Nature then reveals whether a matching session occurs, which happens with probability θ. If it does, in stage 1B each trader submits a demand, the aggregate is published, and each receives their demand net of the average at the stage 1A price. After period 1 trade, the asset liquidates with probability $1-\delta$, and traders receive its value net of a quadratic holding cost on their inventory. Otherwise, at period 2 each trader receives a new inventory shock and signal and trades in a second exchange session, after which the asset liquidates and traders receive its value net of the holding cost on their final positions.

period 2. Upon liquidation in period t, each trader receives the value of their position minus their holding costs, $v_t\, z_{i,t} - \frac{\gamma}{2}\, z_{i,t}^2$.

There are three trading sessions across the two periods. The first is a price-clearing session at the start of period 1, which we call stage 1A. The second is a matching (quantity-clearing) session that may take place immediately after, which we call stage 1B. The third is a second price-clearing session at period 2. Each price-clearing session is modeled as a uniform-price double auction (Klemperer and Meyer, 1989, Kyle, 1989, Vives, 2011). Specifically, each trader submits a demand schedule $D_t^i : \mathbb{R} \to \mathbb{R}$ that is a mapping of price to demand, $p \mapsto D_t^i(p)$, which specifies how many units they wish to buy or sell at each possible price. The market-clearing price, p_t, is the price that sets the net demand to zero,

$$\sum_{i=1}^{N+M} D_t^i(p_t) = 0. \tag{1}$$

After a session, each trader pays the equilibrium price times the amount of the asset they were allocated, $p_t \times D_t^i(p_t)$, and is allocated $D_t^i(p_t)$ units of the asset. If they are selling at the equilibrium price, $D_t^i(p_t) < 0$, then trader i receives the price times the amount of the

asset they sell. Modeling trade as a double auction, rather than through a limit-order book, preserves analytical tractability while capturing the essential mechanism of price impact. After the stage 1A exchange, trader i's inventory is $\hat{z}_1^i = h_1^i + D_1^i(p_1)$.

After the stage 1A exchange clears, nature draws $\zeta \in \{0, 1\}$, with $\Pr(\zeta = 1) = \theta$, independently of all other random variables, and ζ is observed by all traders. If $\zeta = 1$, a fixed-price matching session takes place at stage 1B. Unlike an exchange, the matching session does not discover a price. All quantity traded in the matching session settles at the market clearing stage 1A price, p_1. Importantly, regardless of the quantity demanded or cleared, the price per unit is *fixed* (frozen) at p_1. Each trader submits a single quantity, μ_i, which we refer to as their matching session demand. The allocation mechanism, which can be referred to as proportional rationing, allocates to each trader their demand net of the average demand,

$$D_{SD}^i = \mu_i - \frac{Q}{N+M}. \tag{2}$$

A trader who demands more than the average buys in the matching session, and a trader who demands less than the average sells. By construction, $\sum_j D_{SD}^j = 0$, so the matching session is budget-balanced and requires no outside liquidity. After the matching session, trader i's inventory is $z_1^i = \hat{z}_1^i + D_{SD}^i$. If $\zeta = 0$, no trade occurs at stage 1B, so that $D_{SD}^i \equiv 0$ and $z_1^i = \hat{z}_1^i$. The matching session is a stylized version of size discovery mechanisms, such as workup in the U.S. Treasury market and block-crossing sessions in equity dark pools, which allow large quantities to change hands at a price determined elsewhere (Duffie and Zhu, 2017, Antill and Duffie, 2020). Because the price is fixed at p_1, a trader's matching session demand has no impact on the price at which they trade. All traders can infer aggregate demand Q from their quantity demanded versus allocated, and traders use this to update their beliefs about the fundamental value and aggregate inventory positions, affecting trading strategies in period 2.

If the asset does not liquidate after period 1, each trader is subject to a second inventory shock, h_2^i, the liquidation value of the asset is shocked, and all informed traders receive a new signal, S_2^i. Their inventory before period 2 exchange trade is $\hat{z}_2^i = z_1^i + h_2^i$, and their inventory after it is $z_2^i = \hat{z}_2^i + D_2^i(p_2)$. The asset then liquidates.

Before any trade, each trader's expected utility is given by

$$\mathbb{E}\Big[- p_1\big(D_1^i + D_{SD}^i\big) + (1-\delta)\Big(v_1 z_1^i - \frac{\gamma}{2}\big(z_1^i\big)^2\Big) + \delta\Big(- p_2 D_2^i + v_2 z_2^i - \frac{\gamma}{2}\big(z_2^i\big)^2\Big)\Big]. \tag{3}$$

The first term represents the payment for trade at period 1. Trade in the exchange and in the matching session are paired together, since both settle at p_1. The next term applies if liquidation happens after period 1, which occurs with probability $1-\delta$. The first part gives the liquidation payoff, and the second accounts for the holding cost of the trader's position. The third term is the payoff if liquidation does not happen after period 1 and, therefore, occurs after period 2. Within this third term, the first term represents the payment for trade in the period 2 exchange trade session, the second is the liquidation payoff, and the final term is the holding cost of the trader's final position.

Expanding equation (3), we can express the endogenous inventory process in terms of exogenous inventory shocks and traded quantities. Each trader's optimization problem is therefore

$$\max_{\substack{D_1^i,\, D_2^i:\mathbb{R}\to\mathbb{R} \\ \mu_i\in\mathbb{R}}} \mathbb{E}\Bigg[-p_1\big(D_1^i(p_1)+D_{SD}^i\big)+(1-\delta)\Big(v_1\big(h_1^i+D_1^i(p_1)+D_{SD}^i\big) \\ -\frac{\gamma}{2}\big(h_1^i+D_1^i(p_1)+D_{SD}^i\big)^2\Big)+\delta\Big(-p_2D_2^i(p_2)+v_2\big(h_1^i+D_1^i(p_1)+D_{SD}^i+h_2^i+D_2^i(p_2)\big) \\ -\frac{\gamma}{2}\big(h_1^i+D_1^i(p_1)+D_{SD}^i+h_2^i+D_2^i(p_2)\big)^2\Big)\Bigg], \tag{4}$$

where $D_{SD}^i=\mu_i-Q/(N+M)$ if $\zeta=1$ and $D_{SD}^i=0$ if $\zeta=0$. Traders choose demand schedules, $D_1^i(\cdot)$ and $D_2^i(\cdot)$, in the two price-clearing sessions, which are evaluated at the market clearing prices p_1 and p_2. Traders only choose a single quantity, μ_i, in the matching session, since trade settles at the prevailing exchange trade price p_1. The period 1 demand schedule is chosen before ζ is realized, while the matching session demand and the period 2 demand schedule are chosen after.

In the first trading session, the information set of trader i of an informed trader is $\mathcal{F}_i^{(1),I}=\{S_1^i,h_1^i,p_1\}$, where the price is included because traders submit demand schedules that condition on the clearing price, and $\mathcal{F}_i^{(1),U}=\{h_1^i,p_1\}$ for uninformed traders. At stage 1B, traders only learn whether the matching session takes place, so their information set is $\mathcal{F}_i^{(\mathrm{SD}),\tau}=\mathcal{F}_i^{(1),\tau}$. In particular, traders submit their matching session demand before observing Q. In the second trading session, informed trader i observes $\mathcal{F}_i^{(2),I}=\{S_1^i,S_2^i,h_1^i,h_2^i,p_1,p_2,Q\}$, where Q is included only if the matching session took place, and uninformed trader i observes $\mathcal{F}_i^{(2),U}=\{h_1^i,h_2^i,p_1,p_2,Q\}$. The aggregate matching session demand is therefore a second public signal, alongside the two market-clearing prices.

This will play an important role because traders are trying to learn two dimensions in the model: the asset's value and the distribution of inventory across types. While the asset's liquidating value is common to all, how inventory is spread across the types also affects the market-clearing price and therefore trade terms in the second period.

III. Equilibrium

We study a type-symmetric linear Perfect Bayesian Nash Equilibrium. The linear-quadratic specification implies that traders' optimal demand schedules and matching session demands are linear functions of their information and inventories. We solve for the equilibrium by starting with the period 2 price-clearing session and working backward, first to the matching session and then to the period 1 price-clearing session. At period 1, traders recognize that their demand influences not only the current price, but also their allocation in the matching session, the aggregate statistic Q, and the equilibrium price and demand functions at period 2. At stage 1B, traders recognize that their matching session demand influences the period 2 price through Q. Because traders of the same type are ex-ante identical, the equilibrium coefficients that determine their trading behavior are the same within type, and we denote them with a superscript τ. Individual realized demands, however, still vary with the realization of each trader's signals and inventory shocks. Appendix A contains the full derivations.

A. Trading in the Second Session

Starting from the period 2 price-clearing session, each trader solves the following optimization problem:

$$D_2^i(p) \in \underset{D_2^i}{\operatorname{argmax}}\ \mathbb{E}\Big[-pD_2^i + v_2\big(\hat{z}_2^i + D_2^i\big) - \frac{\gamma}{2}\big(\hat{z}_2^i + D_2^i\big)^2 \ \Big|\ \mathcal{F}_i^{(2),\tau}\Big], \tag{5}$$

subject to the market-clearing condition $D_2^i(p) + \sum_{j\neq i} D_2^j(p) = 0$, where p is the market-clearing price. Each trader can therefore be viewed as trading against a residual demand curve. In equilibrium, they will solve for the optimal demand schedule given their beliefs and conditional on all other traders playing their equilibrium demand schedules. An equilibrium condition will be that their beliefs and actions must be mutually consistent. Define trader i's price impact as $\lambda_2^\tau = \frac{\mathrm{d}p_2}{\mathrm{d}D_2^i}$, which is a constant in equilibrium that we pin down later. Differentiating with respect to D_2^i, and recognizing that the trader's demand moves the

price as they trade against the residual demand curve, the optimal demand schedule satisfies the first-order condition

$$\mathbb{E}\big[v_2 - \gamma\big(\hat{z}_2^i + D_2^i\big) \mid \mathcal{F}_i^{(2),\tau}\big] = p + \lambda_2^\tau D_2^i \tag{6}$$

at each price p. The left-hand side is the expected marginal value of an additional unit of the asset, which is its expected value net of the marginal holding cost. The right-hand side is the marginal cost of buying that unit. The trader pays the price, p, for the additional unit, and because their demand moves the price by λ_2^τ, they also pay more for the D_2^i units they were already demanding. Similarly, a trader who sells an additional unit lowers the price they receive on all of the units they sell. In a competitive market, $\lambda_2^\tau = 0$, and the trader trades until the expected marginal value of the asset equals the price. Under imperfect competition, the second term leads traders to stop short of this position, whether they are buying or selling. Because traders submit demand schedules, (6) holds at every price, and the conditional expectation conditions on the price, since the price carries information about the value and distribution of the asset. Solving (6) for D_2^i, the optimal demand schedule is

$$D_2^i(p) = \chi_2^\tau\left(\frac{1}{\gamma}\big(\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] - p\big) - \hat{z}_2^i\right), \qquad \chi_2^\tau := \frac{\gamma}{\gamma + \lambda_2^\tau}. \tag{7}$$

This expression admits a natural interpretation. First, the term inside the large parentheses is the demand schedule a trader would submit in a competitive market, where price impact is zero. The first part is how much a trader tilts their demand to take advantage of their information, relative to the price, scaled by the marginal cost of holding inventory. They also sell off their current inventory position. The prefactor, χ_2^τ, is always between zero and one, as price impact is weakly positive in equilibrium, and shows how strategic traders scale down their demand as they weigh price impact, λ_2^τ, against their holding cost, γ. In competitive markets, this scaling factor is one. We refer to χ_2^τ as the period 2 trade intensity of type τ. In equilibrium, it is the fraction of the gap between a trader's current and competitive inventory position that the trader closes, and a larger value is associated with more efficient trade. In the literature, this scaling is also known as bid shading.

Importantly, the conditional expectation $\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}]$ is a linear function of both signals, both prices, both inventory shocks, and, if the matching session took place, the aggregate matching session demand Q (see equation (28)). For an informed trader with a perfect signal, the weight on all variables except S_2^i is zero. In contrast, the uninformed trader places weight on each of them. They use the two prices to learn about the value of the asset,

their own inventory shocks to remove the effects of their own orders out of those prices, and, as we will see later, Q to better separate the part of the prices that is driven by inventories from the part driven by the value of the asset. We analyze this last channel, which is what distinguishes our model from a sequence of exchange sessions, in Section IIIE.2.

Given the optimal demand schedules, the market-clearing price is

$$\begin{aligned} p_2 =& \frac{\frac{N}{\gamma+\lambda_2^I}}{\frac{N}{\gamma+\lambda_2^I}+\frac{M}{\gamma+\lambda_2^U}}\left(\frac{1}{N}\sum_{j=1}^{N}\mathbb{E}\big[v_2 \mid \mathcal{F}_j^{(2),I}\big]-\gamma\,\frac{1}{N}\sum_{j=1}^{N}\hat{z}_2^j\right) \\ &+\frac{\frac{M}{\gamma+\lambda_2^U}}{\frac{N}{\gamma+\lambda_2^I}+\frac{M}{\gamma+\lambda_2^U}}\left(\frac{1}{M}\sum_{j=1}^{M}\mathbb{E}\big[v_2 \mid \mathcal{F}_j^{(2),U}\big]-\gamma\,\frac{1}{M}\sum_{j=1}^{M}\hat{z}_2^j\right). \end{aligned} \tag{8}$$

The price is only implicitly defined by (8), since the conditional expectations themselves depend on p_2. The equilibrium price thus represents a weighted average of each type's average valuation of the asset, net of the marginal holding cost of the type's average inventory. Traders who face smaller price impacts receive larger weights in this average because they trade more aggressively and therefore have a stronger effect on prices.

Finally, market clearing implies that the set of price impacts $\{\lambda_2^I, \lambda_2^U\}$ must be consistent with the inverse slope of the residual supply curve faced by each trader.[5] That is, $\lambda_2^\tau = -\big(\sum_{j\neq i}\frac{\partial D_2^j}{\partial p_2}\big)^{-1}$, which for an informed trader is

$$\lambda_2^I = \frac{\gamma}{(N-1)\,\chi_2^I\big(1-c_{p_2,2}^I\big)+M\,\chi_2^U\big(1-c_{p_2,2}^U\big)}, \tag{9}$$

where $c_{p_2,2}^\tau$ is the weight a type τ trader's conditional expectation places on p_2. The expression for λ_2^U is symmetric. The denominator of (9), divided by γ, is the sum of the slopes of the other traders' demand schedules, reflecting how much their total demand falls when the price rises by one unit. Each of these slopes, $\chi_2^\tau\big(1-c_{p_2,2}^\tau\big)/\gamma$, has two parts. The first is the trade intensity, so a trader's price impact is smaller when there are more traders on the other side of their order and when those traders trade more aggressively. Because each trader's trade intensity depends on their own price impact, the price impacts of the two types are determined jointly. The second part reflects learning from the price. A higher price could mean that the value of the asset, v_2, is higher, but it could also mean that other traders have

[5] From the market-clearing condition, demand by trader i at price p must be that $D_t^i + \sum_{j\neq i} D_t^j(p) = 0$. Then, differentiating with respect to the price gives $\frac{\partial D_t^i}{\partial p} + \sum_{j\neq i}\frac{\partial D_t^j(p)}{\partial p} = 0$. Solving for $\frac{\partial p}{\partial D_t^i}$, we get that $\lambda_t^\tau := \frac{\partial p}{\partial D_t^i} = -\big(\sum_{j\neq i}\frac{\partial D_t^j(p)}{\partial p}\big)^{-1}$.

negative inventory positions and are buying to move them back toward zero, which reduces their holding costs. When the price increases, a trader who learns from the price attributes part of the increase to a higher v_2, and so they reduce their demand by less. When adverse selection is greater, traders place more weight on the price when estimating the asset's value, so a higher price makes them less willing to sell, and a buyer must move the price further to fill an order.

The matching session improves liquidity in period 2 through this mechanism. Together with p_1, Q helps the uninformed separate the value of the asset from the inventories of other traders in period 1, so they enter period 2 with less uncertainty about v_2. Because their beliefs are more precise, a change in p_2 moves their valuation of the asset by less. They therefore are willing to sell more to a trader whose order pushes up the price, and price impact in period 2 falls.

B. The Matching Session

If $\zeta = 1$, a matching session occurs and each trader demands μ_i given their information set $\mathcal{F}_i^{(\mathrm{SD}),\tau} = \mathcal{F}_i^{(1),\tau}$. The matching session demand has no impact on p_1. The matching session demand affects the period 2 environment in two ways: it alters the inventory the trader carries into period 2, and it alters the aggregate demand Q that the other traders use to update their beliefs. Therefore, the demanded quantity in period 1B will have a cross-period price impact on the clearing price in period 2. Denote the resulting cross-period price impact by $\lambda_{\mathrm{SD},2}^{\tau} = \frac{\mathrm{d}p_2}{\mathrm{d}\mu_i}$. Each trader's optimization problem at stage 1B is

$$\mu_i \in \underset{\mu_i}{\operatorname{argmax}}\ \mathbb{E}\Big[-p_1 D_{SD}^i + (1-\delta)\Big(v_1\big(\hat{z}_1^i + D_{SD}^i\big) - \frac{\gamma}{2}\big(\hat{z}_1^i + D_{SD}^i\big)^2\Big) + \delta\, V_{i,2} \;\Big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\Big], \quad (10)$$

where $V_{i,2} = -p_2 D_2^i + v_2 z_2^i - \frac{\gamma}{2}(z_2^i)^2$ is the period 2 continuation value. After some simplification, the first-order condition can be written as

$$\mathbb{E}\big[v_1 - \gamma\big(\hat{z}_1^i + D_{SD}^i\big) \;\big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\big] = \Omega_{\mathrm{SD}}^{\tau}\, p_1 + \big(1 - \Omega_{\mathrm{SD}}^{\tau}\big)\, \mathbb{E}\big[p_2 \;\big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\big], \quad (11)$$

where

$$\Omega_{\mathrm{SD}}^{\tau} = \frac{1}{1 - \delta\big[1 - (1-\chi_2^{\tau})^2\big] - \delta\,(1-\chi_2^{\tau})(2-\chi_2^{\tau})\,\dfrac{(N+M)\,\lambda_{\mathrm{SD},2}^{\tau}}{(N+M-1)\,\lambda_2^{\tau}}}. \quad (12)$$

The left-hand side of (11) is the expected value of the asset net of the marginal holding cost. The right-hand side is an effective price, a weighted combination of the price at which the

unit is bought, p_1, and the price the trader expects at period 2. In a competitive market, $\chi_2^\tau = 1$ and $\Omega_{\text{SD}}^\tau = \frac{1}{1-\delta}$, so the effective price is $\frac{p_1 - \delta\,\mathbb{E}[p_2]}{1-\delta}$, the cost of a unit bought at p_1 and resold at p_2 if the asset does not liquidate. Under imperfect competition, Ω_{SD}^τ deviates from $\frac{1}{1-\delta}$ for two reasons. First, the trader will only partially unwind their position at period 2 due to bid shading in response to the price impact, since $\chi_2^\tau < 1$. Second, the trader's matching session demand moves the period 2 price through the cross-period price impact, $\lambda_{\text{SD},2}^\tau$.

Solving (11) for the matching session allocation, the expected allocation can be written as the expected competitive allocation plus an additional term due to imperfect competition,

$$\mathbb{E}\big[D_{SD}^i \;\big|\; \mathcal{F}_i^{(\text{SD}),\tau}\big] = \underbrace{\frac{1}{\gamma}\left(\mathbb{E}[v_1 \mid \mathcal{F}_i^{(\text{SD}),\tau}] - \frac{p_1 - \delta\,\mathbb{E}[p_2 \mid \mathcal{F}_i^{(\text{SD}),\tau}]}{1-\delta}\right) - \hat{z}_1^i}_{\text{competitive demand}} + \underbrace{\frac{1}{\gamma}\Big(\frac{1}{1-\delta} - \Omega_{\text{SD}}^\tau\Big)\big(p_1 - \mathbb{E}[p_2 \mid \mathcal{F}_i^{(\text{SD}),\tau}]\big)}_{\text{imperfect-competition correction}}. \tag{13}$$

Unlike the exchange demand in (7), the competitive part of the matching session allocation is not scaled down by a trade intensity. The trader's demand does not move the price at which they trade, so they aim for their full desired position given the effective price. Only the correction for the cross-period price impact in period 2 remains. Since the allocation is the trader's demand net of the average demand, the matching session demand itself satisfies

$$\frac{N+M-1}{N+M}\,\mu_i = \frac{1}{\gamma}\Big(\mathbb{E}[v_1 \mid \mathcal{F}_i^{(\text{SD}),\tau}] - \big(\Omega_{\text{SD}}^\tau\, p_1 + \big(1 - \Omega_{\text{SD}}^\tau\big)\big)\,\mathbb{E}[p_2 \mid \mathcal{F}_i^{(\text{SD}),\tau}]\Big) - \hat{z}_1^i + \mathbb{E}\left[\frac{1}{N+M}\sum_{j\neq i}\mu_j \;\middle|\; \mathcal{F}_i^{(\text{SD}),\tau}\right]. \tag{14}$$

The last term is the trader's forecast of the average demand of their rivals. A trader who expects their rivals to demand a lot must demand more themselves to receive the same allocation. Two features therefore distinguish a matching session demand from a demand schedule in a price-clearing session. First, there is no trade intensity scaling down the demand. Because trade settles at p_1, the trader's matching session demand does not move the price at which they trade. Imperfect competition still affects this target, but only through period 2. The weight Ω_{SD}^τ depends on the period 2 trade intensity and on how the matching session demand moves the period 2 price through a cross-period price impact, $\lambda_{\text{SD},2}^\tau$. Second,

because each trader's allocation is their demand net of the average demand, the demand contains the trader's forecast of their rivals' matching session demands. Both features are central to why and about what Q is informative, which we discuss in Section IIIE.2.[6]

C. Trading in the First Session

In period 1A, traders submit demand schedules before learning whether the matching session will take place.[7] The decision at date 1 depends on two "branches" of the model, which are characterized by the realization of ζ. We first describe the period 1 first-order condition on each branch separately, and then combine them. In each case, traders act strategically, recognizing that their current demand influences not only the current price but also future prices and demands. Importantly, because of the linear-quadratic setting we consider, these marginal impacts are constant, keeping this non-Markovian model tractable. We denote the cross-period price impact by $\lambda^{\tau}_{1,2} = \frac{\mathrm{d}p_2}{\mathrm{d}D^i_1}$, the cross-period demand impact by $\frac{\mathrm{d}D^i_2}{\mathrm{d}D^i_1}$, and the trader's current price impact by $\lambda^{\tau}_1 = \frac{\mathrm{d}p_1}{\mathrm{d}D^i_1}$.

No matching session. Suppose first that there is no matching session. Each trader's optimization problem at period 1 is then

$$D^i_1(p) \in \underset{D^i_1}{\operatorname{argmax}}\ \mathbb{E}\Big[-pD^i_1 + (1-\delta)\Big(v_1\big(h^i_1 + D^i_1\big) - \frac{\gamma}{2}\big(h^i_1 + D^i_1\big)^2\Big) + \delta\, V_{i,2} \ \Big|\ \mathcal{F}^{(1),\tau}_i\Big], \quad (15)$$

where $V_{i,2}$ is the period 2 continuation value defined in Section IIIB. Unlike at period 2, the trader's demand now also affects this continuation value. An additional unit bought at period 1 is carried into period 2, where the trader sells part of it. Implicitly differentiating

[6] Matching session demands are determined only up to a common shift that depends on public information. If every trader adds the same amount $f(p_1)$ to their demand, then each allocation D^i_{SD} is unchanged and Q shifts by the publicly known amount $(N+M)\,f(p_1)$, so it carries the same information. Allocations, prices, price impacts, beliefs, and welfare are therefore the same for every such shift. Within the linear equilibria we study, $f(p_1) = k_0 + k\,p_1$ for constants k_0 and k. We set $k_0 = 0$, since all random variables have mean zero. Only the loadings of μ_i and of period 2 demand on p_1, and the variance of Q, depend on k, and we report them for a single normalization, in which we choose k so that the informed's matching session demand places no weight on p_1.

[7] If the matching session took place for certain, $\theta = 1$, units bought in the price-clearing session and in the matching session both settle at p_1, so they are perfect substitutes. Trading in the price-clearing session creates price impact, which the matching session avoids, so traders would shift their trade to the matching session. The price-clearing session would then no longer determine a price, and the matching session would have no price at which to trade. We therefore focus on $\theta \in [0, 1)$.

the period 2 first-order condition with respect to period 1 demand,

$$\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} = -\frac{\gamma + \lambda_{1,2}^\tau}{\gamma + \lambda_2^\tau}. \tag{16}$$

The amount the trader sells at period 2 depends on the period 2 holding cost, the period 2 price impact, and how the period 1 trade affects the period 2 price. The cross-period price impact, $\lambda_{1,2}^\tau$, is determined by differentiating period 2 market clearing with respect to period 1 demand (Appendix AE.4). It captures that a period 1 trade changes the inventories that the trader and their counterparties carry into period 2, as well as the period 1 price, which rivals use to form their period 2 beliefs.

After some simplification, the period 1 first-order condition can be written as

$$\kappa_v^\tau \,\mathbb{E}\big[v_1 - \gamma\big(h_1^i + D_1^i\big) \;\big|\; \mathcal{F}_i^{(1),\tau}\big] + \big(1 - \kappa_v^\tau\big)\,\mathbb{E}\big[p_2 \;\big|\; \mathcal{F}_i^{(1),\tau}\big] = p + \lambda_1^\tau D_1^i, \tag{17}$$

where

$$\kappa_v^\tau := (1-\delta) + \delta\,(1-\chi_2^\tau)\left(1 + \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} - \frac{\lambda_{1,2}^\tau}{\lambda_2^\tau}\right). \tag{18}$$

The right-hand side of (17) is the marginal cost of the purchase of a unit, given by the price, p, plus the increase in the price paid on the trader's other units, $\lambda_1^\tau D_1^i$. The left-hand side is the marginal benefit, a weighted average of the unit's marginal value after period 1 and the price the trader expects at period 2. In a competitive market, $\kappa_v^\tau = 1 - \delta$. With probability $1 - \delta$, the asset liquidates and the unit is worth its marginal value, and with probability δ, the trader resells the unit at p_2. Under imperfect competition, κ_v^τ also depends on how much of the unit the trader unwinds at period 2 and on how their period 1 demand affects the period 2 price. Solving (17) for D_1^i, the period 1 demand schedule is

$$D_1^i(p) = \chi_1^\tau\left(\frac{1}{\gamma}\left(\mathbb{E}\big[v_1 \;\big|\; \mathcal{F}_i^{(1),\tau}\big] - \frac{p - \big(1-\kappa_v^\tau\big)\,\mathbb{E}\big[p_2 \;\big|\; \mathcal{F}_i^{(1),\tau}\big]}{\kappa_v^\tau}\right) - h_1^i\right), \tag{19}$$

where $\chi_1^\tau := \frac{\gamma\kappa_v^\tau}{\gamma\kappa_v^\tau + \lambda_1^\tau}$. This form is analogous to the period 2 demand in equation (7). The term inside the large parentheses is the competitive demand at an effective price that combines p and the expected period 2 price. The outer fraction, χ_1^τ, is the period 1 trade intensity, which moderates how aggressively the trader trades. Without price impact, $\kappa_v^\tau = 1 - \delta$, $\chi_1^\tau = 1$, and the effective price is $\frac{p - \delta\,\mathbb{E}[p_2]}{1-\delta}$, as in the matching session.

Both informed and uninformed traders form conditional expectations that depend on their signal, if they receive one, their inventory shock, and the market-clearing price. The uninformed traders infer information about both the fundamental value and the distribution

of inventories across trader types from the observed price. In contrast, informed traders with signals use the price disproportionately more to update their beliefs about the inventories of other traders. These beliefs inform traders' expectations of the price that they will encounter in the second trading session.

Probabilistic matching session When the matching session takes place with probability θ, the period 1 first-order condition weights the marginal expected utilities on the two branches by $1-\theta$ and θ,

$$(1-\theta)\Big(\kappa_v^\tau\,\mathbb{E}\big[v_1-\gamma\big(h_1^i+D_1^i\big)\;\big|\;\mathcal{F}_i^{(1),\tau}\big]+\big(1-\kappa_v^\tau\big)\,\mathbb{E}\big[p_2\;\big|\;\mathcal{F}_i^{(1),\tau},\zeta=0\big]-p_1-\lambda_1^\tau D_1^i\Big)+\theta\,G_{\mathrm{SD}}^i=0, \tag{20}$$

where the first term is the marginal expected utility from (17) and G_{SD}^i is the marginal expected utility on the matching session branch, derived in Appendix AD. The second term does not depend on D_1^i. On the matching session branch, the trader re-optimizes their position in the matching session, so an additional unit bought in the price-clearing session can be sold back in the matching session. The session in which the trader purchases a unit is not irrelevant. A unit purchased in the price-clearing session affects the period 2 price in a different way than a unit purchased in the matching session, and it increases p_1, which applies to all of the trader's period 1 trade. Period 1 demand is determined by the first term alone, which receives weight $1-\theta$. Solving (20) for D_1^i, the resulting demand schedule is

$$D_1^i(p_1)=\frac{\chi_1^\tau-\theta}{1-\theta}\left(\frac{1}{\gamma}\Big(\mathbb{E}\big[v_1\;\big|\;\mathcal{F}_i^{(1),\tau}\big]-\bar{p}_1^{\,i}\Big)-h_1^i\right), \tag{21}$$

where $\bar{p}_1^{\,i}$ is an effective price that combines p_1 and the trader's forecasts of p_2 on both of the two branches (Appendix AD). This is of the same form as (19), but the anticipation of the matching session lowers the period 1 trade intensity from χ_1^τ to $\frac{\chi_1^\tau-\theta}{1-\theta}$.[8] With probability θ, a unit bought in the price-clearing session can be undone in the matching session, so it only adds price impact. Traders postpone part of their inventory adjustment to the matching session, even though it takes place only with probability θ, which harms liquidity in the period 1 price-clearing session (Antill and Duffie, 2020). As $\theta\to 1$, the branch without a matching session receives no weight, and the period 1 demand schedule is no longer determined, which is the degeneracy described above. When $\theta=0$, (21) reduces to (19).

[8] This requires $\theta<\chi_1^\tau$, which holds in every parameterization we study.

D. *Equilibrium*

We solve for a type-symmetric linear Perfect Bayesian equilibrium, which is characterized by the following proposition:

PROPOSITION 1: *A Perfect Bayesian linear equilibrium requires that:*

1. *Given the price impacts* λ_1^τ, λ_2^τ, $\lambda_{1,2}^\tau$, *and* $\lambda_{\mathrm{SD},2}^\tau$, *and the cross-period demand impacts* $\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}$, $\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}$, *and* $\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}$, *exchange demands and matching session demands are optimal given individuals' information sets, and exchange prices are chosen to set aggregate demands to 0,*
2. *Given the price and demand impacts and equilibrium demands and prices, beliefs* $\mathbb{E}[v_1 \mid \mathcal{F}_i^{(1),\tau}]$, $\mathbb{E}[p_2 \mid \mathcal{F}_i^{(1),\tau}]$, $\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}]$, *and the forecasts of rivals' matching session demands,* $\mathbb{E}\big[\frac{1}{N+M}\sum_{j\neq i}\mu_j \mid \mathcal{F}_i^{(\mathrm{SD}),\tau}\big]$, *are determined by Bayesian updating,*
3. $\{\lambda_1^\tau, \lambda_2^\tau\}$ *are determined to ensure price impacts are consistent with residual demand curves, while* $\{\lambda_{1,2}^\tau, \lambda_{\mathrm{SD},2}^\tau\}$ *and the cross-period demand impacts are determined by implicit differentiation of the period 2 market-clearing condition and of the period 2 and matching session first-order conditions.*

This characterization is an application of Rostek et al. (2025) to the case in which a matching session is modeled. The model quickly becomes complex because a trader's equilibrium price impact depends not only on how traders of the same type react and learn, but also on the responses of traders of the other type, both in the current and in future sessions. The matching session adds to this complexity in three ways. It creates a second branch at period 2, on which traders observe aggregate quantity demanded, Q, and hold different inventories. It creates a new cross-period price impact, $\lambda_{\mathrm{SD},2}^\tau$. And, it requires each trader to forecast the demands of their rivals, because their allocation depends on the realization of Q, which is not observed when the demands are submitted. With two trader types, the following twelve price impacts are jointly determined in equilibrium:

$$\big\{\lambda_1^\tau,\ \lambda_2^{\tau,\mathrm{SD}},\ \lambda_2^{\tau,\mathrm{no}},\ \lambda_{1,2}^{\tau,\mathrm{SD}},\ \lambda_{1,2}^{\tau,\mathrm{no}},\ \lambda_{\mathrm{SD},2}^\tau\big\}_{\tau\in\{I,U\}}, \tag{22}$$

where the superscripts SD and no denote the matching session ($\zeta = 1$) and no matching session ($\zeta = 0$) branches. The period 1 price impact is common to both branches, since period 1 demand is submitted before ζ is realized. This intertemporal and cross-type dependence creates a nonlinear, non-recursive fixed-point problem for the price impacts, the strategy

coefficients, and the projection coefficients. As a result, we must rely on numerical methods to solve the model (Appendix AH). When $\theta = 0$, the model reduces to a two-period model of Rostek et al. (2025), which we use as a benchmark throughout.

E. Equilibrium Properties

We now illustrate the economic forces in the model. Unless otherwise stated, we use the following parameterization throughout:

$$N = 20, \quad M = 20, \quad \gamma = 2, \quad \delta = 0.5, \quad \theta = 0.1,$$
$$\sigma^2 = 1, \quad \sigma_\eta^2 = 0.1, \quad \sigma_{\epsilon,I}^2 = .25, \quad \sigma_h^2 = 1.$$

Setting $\sigma_{\epsilon,I}^2 = 0.25$ means that informed traders observe a noisy signal of v_t at each period, while uninformed traders have no private signal and learn about the value of the asset only from the prices and, when the matching session takes place, from Q. The value $\sigma^2 = 1$ normalizes the prior variance of the value of the asset, and $\sigma_\eta^2 = 0.1$ ensures that the period 2 signals carry meaningful new information. With $\delta = 0.5$, the asset liquidates with probability one half between the two periods, and $\theta = 0.1$ means the matching session happens 10% of the time. The market has twenty traders of each type, and inventory shocks have the same variance as the value of the asset, $\sigma_h^2 = \sigma^2 = 1$. We compare the equilibrium to the benchmark without a matching session, in which $\theta = 0$.

Table I reports the equilibrium demand coefficients in an economy with and without the matching session. Informed traders trade on their signal, while the uninformed, who have no private signal, place no weight on it. Both types sell off most of their inventory shock in period 1. Anticipating the matching session, traders postpone part of their inventory adjustment to it, as described in Section IIIC, resulting in the loadings on the inventory shock being slightly smaller in magnitude for both types.

The matching session demand loads negatively on the inventory shock, so a trader who receives a larger inventory shock submits a lower matching session demand. Because each trader receives their demand net of the average demand, traders with above-average inventories sell in the matching session, and traders with below-average inventories buy. The matching session therefore moves traders part of the way toward the cross-sectional average inventory. For the informed, the loading of -0.1205 is close to the $1 + (-0.8645) = 0.1355$ units of each inventory shock that remain after the period 1 exchange. Because the price is

Table IEquilibrium Demand Coefficients
We report the loadings of the period 1 demand, the matching session demand, and the period 2 demand on each element of the trader's information set, by type. The columns labeled "Matching" report the equilibrium with a probabilistic matching session, $\theta = 0.1$, with the matching session demand and period 2 demand evaluated on the matching session branch ($\zeta = 1$). The columns labeled "None" report the benchmark without a matching session, $\theta = 0$. The matching session demand is written in terms of (S_1^i, h_1^i, p_1). Its loading on p_1 and the matching session branch period 2 loading on p_1 are reported for the normalization described in Section IIIB. The table uses the baseline parameterization.

	Informed		Uninformed	
Loading	Matching	None	Matching	None
Period 1 demand, D_1^i				
S_1^i	+0.3138	+0.3080	—	—
h_1^i	−0.8645	−0.8780	−0.8435	−0.8566
p_1	−0.3422	−0.3398	−0.0903	−0.0942
Matching session demand, μ_i				
S_1^i	+0.0161	—	—	—
h_1^i	−0.1205	—	−0.1161	—
p_1	+0.0000	—	+0.0133	—
Period 2 demand, D_2^i				
S_1^i	−0.3201	−0.1456	—	—
S_2^i	+0.1477	+0.2413	—	—
p_1	+1.1182	+0.3389	+1.1970	+0.2901
p_2	−0.4373	−0.4244	−0.4127	−0.3013
h_1^i	−0.0151	−0.1112	−0.0385	−0.1020
h_2^i	−0.9680	−0.9618	−0.9666	−0.9541
Q	−0.5297	—	−0.7678	—

fixed, the informed use the matching session to shed most of their remaining excess inventory, as in Antill and Duffie (2020) and Blonien (2026), where matching sessions perfectly reallocate the excess inventory. Notice, however, that the loading on their private signal is fairly low, especially relative to the loading in period 1A. As we will see, the cross-period demand impact from the matching session and period 2 price will be substantial, to discipline informed traders from trading too aggressively on their private information.

The matching session also changes period 2 demand. Without it, period 2 demand places

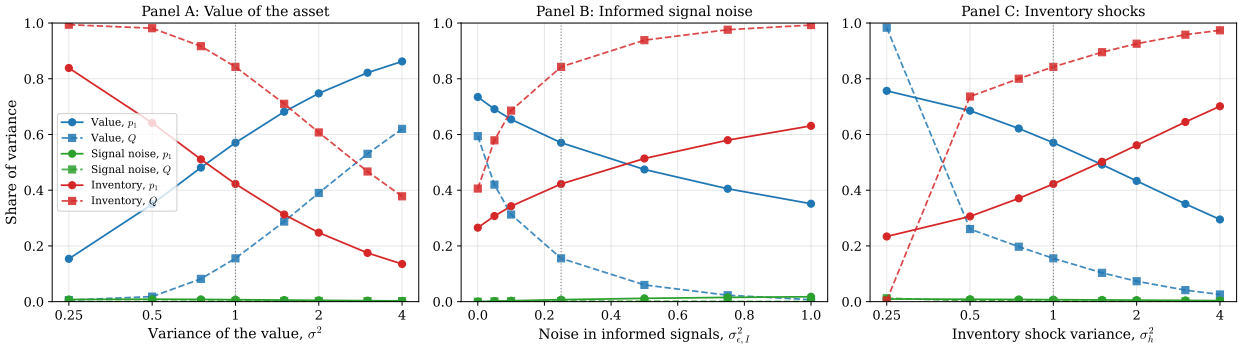

Figure 2. Variance Decomposition of the Public Signals
We plot the share of the variance of the period 1 price, p_1 (solid), and of the aggregate matching session demand, Q (dashed), attributable to the value of the asset, v_1, to the noise in traders' signals, $\{\epsilon_{i,1}\}_i$, and to traders' inventory shocks, $\{h_1^i\}_i$. Panel A varies the variance of the value of the asset, σ^2, Panel B varies the noise in the informed's signals, $\sigma^2_{\epsilon,I}$, and Panel C varies the variance of inventory shocks, σ^2_h. Dotted vertical lines mark the baseline parameterization, and all other parameters are at their baseline values. For Q, we decompose the part that loads on traders' signals and inventory shocks, which does not depend on the normalization described in Section IIIB.

substantial weight on the period 1 inventory shock, as traders finish offloading positions they did not offload at period 1. With it, that weight is close to zero, since the matching session has already offloaded most of traders' excess inventory. At period 2, the uninformed also place a large negative weight on Q. A high aggregate matching session demand indicates that traders are, on average, short inventory. Part of the period 1 price therefore reflected buying pressure from inventories rather than the value of the asset, and the uninformed lower their valuation. Because the signals of the informed are noisy in the baseline parameterization, the informed also learn from Q, and they place a negative weight on it as well. If the informed observed the value of the asset exactly, they would have nothing to learn from Q. Their period 2 demand would still depend on Q, but only because their allocation in the matching session, and therefore the inventory they carry into period 2, depends on it.

Figure 2 shows that variation in the aggregate matching session demand Q is primarily due to inventory shocks, while the price in period 1A is more of a function of the private signals in the aggregate. At the benchmark parameterization, inventory shocks account for 42% of the variance of the period 1 price, but 84% of the variance of Q.

Table IIEquilibrium Price Impacts
We report the own-period and cross-period price impacts, by type. λ_1^τ is the period 1 price impact, which is common to both branches. $\lambda_2^{\tau,\mathrm{SD}}$ and $\lambda_2^{\tau,\mathrm{no}}$ are the period 2 price impacts on the matching session and no matching session branches, and $\lambda_{1,2}^{\tau,\mathrm{SD}}$ and $\lambda_{1,2}^{\tau,\mathrm{no}}$ are the corresponding cross-period price impacts of period 1 demand on the period 2 price. $\lambda_{\mathrm{SD},2}^\tau$ is the cross-period price impact of the matching session demand on the period 2 price. The column labeled "None" reports the benchmark without a matching session, $\theta = 0$. The table uses the baseline parameterization.

Price impact	Matching ($\theta = 0.1$)	None ($\theta = 0$)	Difference
λ_1^I	0.1204	0.1199	+0.0005
λ_1^U	0.1168	0.1165	+0.0003
$\lambda_2^{I,\mathrm{SD}}$	0.0604	0.0710	−0.0106
$\lambda_2^{U,\mathrm{SD}}$	0.0603	0.0704	−0.0101
$\lambda_2^{I,\mathrm{no}}$	0.0690	0.0710	−0.0020
$\lambda_2^{U,\mathrm{no}}$	0.0685	0.0704	−0.0019
$\lambda_{1,2}^{I,\mathrm{SD}}$	+1.7465	+0.0345	+1.7120
$\lambda_{1,2}^{U,\mathrm{SD}}$	+1.7175	+0.0317	+1.6858
$\lambda_{1,2}^{I,\mathrm{no}}$	+0.0375	+0.0345	+0.0031
$\lambda_{1,2}^{U,\mathrm{no}}$	+0.0346	+0.0317	+0.0029
$\lambda_{\mathrm{SD},2}^I$	−1.5466	—	—
$\lambda_{\mathrm{SD},2}^U$	−1.5305	—	—

E.1. Price Impacts

Table II reports the price impacts with and without the matching session. Period 1 price impact is less than 1% higher when the matching session is available. Anticipating the matching session, traders shade their period 1 demand schedules, but with forty traders the period 1 exchange is fairly liquid, and the bid shading is small in magnitude. Period 2 price impact is lower on both branches than in the benchmark without a matching session. With a matching session, traders learn from Q, which allows the uninformed to better disentangle the source of period 2 demand, and they trade more aggressively. Period 2 price impact is also slightly lower even without a matching session. Anticipating the matching session, traders postpone part of their inventory-driven trade, but not their information-driven trade. The informed's ratio of signal to inventory loadings at period 1 rises, from 0.35 to 0.36, so the period 1 price is more informative about the value of the asset, and the uninformed face

less adverse selection at period 2.

The cross-period price impacts are much larger when there is a matching session. The impact of the matching session demand on the period 2 price, $\lambda^{\tau}_{\text{SD},2}$, is about 25 times as large as the period 2 price impact itself. Both result from the information content of Q for the uninformed. A trader who increases their matching session demand increases Q. The uninformed interpret the higher aggregate demand as evidence that traders are short inventory, lower their valuation of the asset, and sell at period 2, which lowers the period 2 price. The trader also carries more inventory into period 2, which they sell as well. The cross-period price impact of period 1 demand, $\lambda^{\tau}_{1,2}$, is also large when there is a matching session. Suppose a trader buys an additional unit at period 1, which increases p_1. Without Q, the uninformed cannot discern whether a higher p_1 reflects a higher value of the asset or traders who are short inventory, so they only partly attribute the price change to the value of the asset. Knowing that with Q, they can tell the two apart, they attribute more of the higher price to the value of the asset. They raise their valuation and buy at period 2, which increases p_2. The effect is even larger because of the matching session. After buying the additional unit, the trader holds more inventory, and a trader with more inventory demands less in the matching session, so Q falls. The uninformed interpret the lower Q as a signal that traders hold more inventory than they thought. More inventory should push the period 1 price down, yet p_1 went up. The uninformed conclude that the value of the asset must be even higher, which increases p_2 further. Without a matching session, cross-period price impacts are small.

Because trade in the matching session settles at a fixed price, the price impact of the matching session demand on the period 2 price, $\lambda^{\tau}_{\text{SD},2}$, is the only price impact a trader faces there. Beyond the holding cost of the inventory they receive, it is the only thing that restrains an aggressive matching session demand. This deterrent falls mostly on the informed when they trade on their signals. The informed unwind part of their signal-driven position at period 2, so a matching session demand that moves the period 2 price moves it against their period 2 trade. In contrast, after the matching session, period 2 demand places almost no weight on the period 1 inventory shock, so the deterrent has little effect on inventory-driven demand. In unreported results, if traders ignored this cross-period price impact when choosing their matching session demand, the informed would place about two and a half times as much weight on their signal. The price impact of the matching session demand

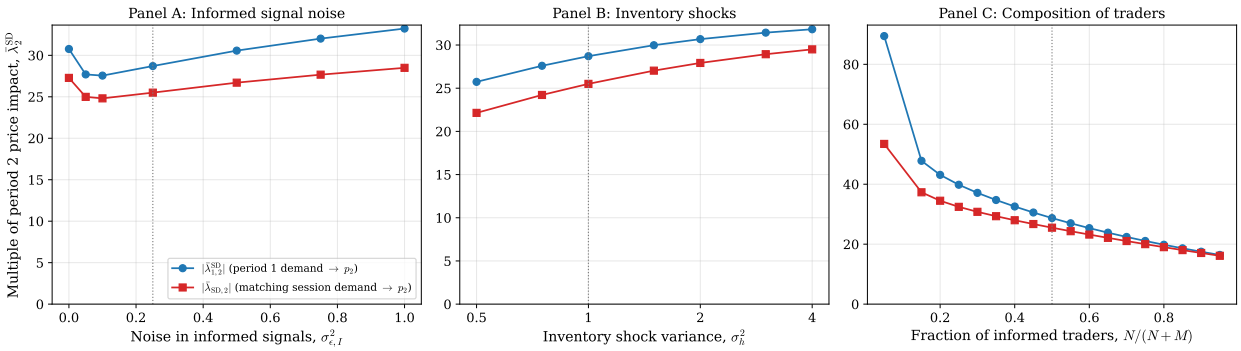

Figure 3. Cross-Period Price Impacts
We plot the population-weighted cross-period price impacts on the matching session branch as multiples of the population-weighted period 2 price impact, $\bar{\lambda}_2^{SD}$, with $\theta = 0.1$. The blue line reports the price impact of period 1 demand on the period 2 price, $|\bar{\lambda}_{1,2}^{SD}|$, and the red line reports the price impact of the matching session demand on the period 2 price, $|\bar{\lambda}_{SD,2}|$. Population weights are $N/(N+M)$ for informed and $M/(N+M)$ for uninformed traders. Panel A varies the noise in the informed's signals, $\sigma^2_{\epsilon,I}$, Panel B varies the variance of inventory shocks, σ^2_h, and Panel C varies the fraction of informed traders, $N/(N+M)$, holding $N+M=40$ fixed. Dotted vertical lines mark the baseline parameterization, and all other parameters are at their baseline values.

and the information in Q therefore reinforce each other. Because Q is mostly a signal about inventories, a deviation in the matching session moves beliefs and the period 2 price by a lot, and because it does, the informed trade little on their signals there, which keeps Q a signal about inventories.

Figure 3 shows how the cross-period price impacts vary with the parameters of the model. We scale each by the period 2 price impact, so that it measures how much more a deviation moves the period 2 price through the beliefs of other traders than a period 2 trade does. In every parameterization, both cross-period price impacts are several times as large as the period 2 price impact, so the deterrent against deviating in the matching session is large. The size of the deterrent depends on how much the uninformed rely on the public signal. Apart from when the informed's signals are nearly exact, both cross-period price impacts increase with the noise in the informed's signals. When the informed know less, they also learn from Q and p_1, so more traders react to any deviation. Both decrease with the fraction of informed traders, since there are fewer uninformed traders whose period 2 demands respond

to Q and p_1. With two informed traders, the cross-period price impact of period 1 demand is about 90 times the period 2 price impact. Larger inventory shocks increase the cross-period price impacts. The price impact of the matching session demand increases with the variance of inventory shocks, since Q is a signal about inventories and the uninformed place more weight on it when inventory shocks are larger. The price impact of period 1 demand is nearly unchanged, but the period 2 price impact falls, since more of period 2 trade is driven by inventories rather than information.

When the cross-period price impact of period 1 demand is large, traders shade their period 1 demand more in anticipation of the matching session. As Table IV shows, the increase in period 1 price impact from the matching session is largest when there are few informed traders, which coincides with when the cross-period price impact of period 1 demand is largest. Cross-period price impact reduces aggressiveness in period 1, which increases price impact during period 1 itself. However, with forty traders, the increase is small in every parameterization.

E.2. Why the Matching Session Is Informative

The period 1 price combines two sources of information: the value of the asset and the inventories of other traders. To see this, consider the case in which informed traders observe the fundamental value exactly, $\sigma^2_{\epsilon,I} = 0$, and uninformed traders have no private signal, and write the period 1 demand as $D^i_1 = a^\tau_{1,S} S^i_1 + a^\tau_{1,h} h^i_1 + a^\tau_{1,p} p_1$. Market clearing then gives

$$p_1 = \ell_v \, v_1 + \ell_I \, H_I + \ell_U \, H_U, \qquad \ell_v := \frac{N \, a^I_{1,S}}{A}, \qquad \ell_\tau := \frac{a^\tau_{1,h}}{A}, \tag{23}$$

where $H_\tau = \sum_{j \in \tau} h^j_1$ is the aggregate inventory shock of type τ and $A := -\big(N a^I_{1,p} + M a^U_{1,p}\big) > 0$ is the aggregate slope of period 1 demand. In equilibrium, demand increases in the informed's signal and decreases in each trader's inventory shock, as the period 1 demand schedule (19) suggests and Table I confirms, so that $\ell_v > 0$ and $\ell_\tau < 0$. An uninformed trader who observes a low price cannot discern whether the value of the asset is low or whether other traders hold large inventories. A second price-clearing session would not help to separate the components of the price, since its price aggregates traders' valuations in the same way and so reveals nearly the same combination of fundamental value and inventories.[9]

[9] In a numerical counterfactual in which the matching session is replaced by a second uniform-price double auction at stage 1B, the two prices are nearly collinear, and a linear equilibrium fails to exist unless new

The matching session's demands, on the other hand, weight value and inventory differently from the price. Let m_S^τ and m_h^τ denote the loadings of the matching session demand on the signal and on the inventory shock, as reported in Table I. Removing the part of Q that is already known from the price, the new information in Q is

$$Q^\perp = r_I\, H_I + r_U\, H_U, \qquad r_\tau := m_h^\tau - \frac{m_S^I}{a_{1,S}^I}\, a_{1,h}^\tau. \tag{24}$$

Given the price, Q is a pure inventory signal. This calculation requires only public information and knowledge of the equilibrium strategies. If matching session demands were simply rescaled copies of period 1 demands, then $r_\tau = 0$ and Q would reveal no new information. Matching session demands instead tilt toward inventory for two reasons. First, because trade settles at a fixed price, traders use the matching session to shed most of the inventory left after period 1, while their signal enters only through the gap between their bid-shaded period 1 demand and their desired position. Second, as discussed in Section IIIE.1, the price impact of the matching session demand deters the informed from trading on their signals. Without this deterrent, the informed would place about two and a half times as much weight on their signal in the matching session.

The uninformed therefore have two public signals and three unknowns. Q tells them about the inventories of other traders, and once they account for those inventories, the period 1 price tells them about the value of the asset. Revelation would be exact if the two types loaded on their inventories in the same way, $r_I/r_U = \ell_I/\ell_U$. Because the types load somewhat differently, some uncertainty remains, but the matching session removes most of it: at the baseline, the conditional variance of the period 2 value for the uninformed falls from 0.51 to 0.11 (Table III). In this special case, only the uninformed learn, since the informed already know the value. In the baseline parameterization, the signals of the informed are noisy, so the informed also learn from Q, as the negative weight on Q in Table I shows.

inventory shocks arrive between the two sessions. When it exists, the second price is nearly uninformative.

IV. Welfare and the Informational Role of the Matching Session

A. *Welfare*

We now analyze the welfare implications of the model. Specifically, we define aggregate welfare as the sum of traders' ex-ante expected payoffs. Because payments for trade sum to zero across traders by market clearing and budget balance, and because $\sum_i z_1^i = \sum_i h_1^i$ and $\sum_i z_2^i = \sum_i (h_1^i + h_2^i)$ are independent of the value of the asset, neither payments nor the liquidation value of positions contribute to aggregate welfare. Aggregate welfare is therefore the negative of expected holding costs. Aggregate welfare, normalized by the total number of traders, is therefore

$$W = -\frac{\gamma}{2}\left(\frac{N}{N+M}\times\mathbb{E}_I\left[(1-\delta)\left(z_1^i\right)^2+\delta\left(z_2^i\right)^2\right]+\frac{M}{N+M}\times\mathbb{E}_U\left[(1-\delta)\left(z_1^i\right)^2+\delta\left(z_2^i\right)^2\right]\right)]. \quad (25)$$

Since traders within a type are ex-ante identical, the efficient allocation splits the aggregate inventory equally, and welfare is higher when inventories are more evenly distributed across traders at the end of each period. With a probabilistic matching session, $W = (1-\theta)\, W^{\text{no}} + \theta\, W^{\text{SD}}$, where W^{SD} and W^{no} denote welfare on the matching session and no matching session branches. Payments net to zero only in the aggregate. Within a type, expected payments need not be zero, since informed traders trade against uninformed traders at favorable prices. We therefore define the welfare of a trader of type τ, W^τ, as their ex-ante expected payoff in (3), which includes their transfers from trading and the liquidation values of their positions.

The matching session affects welfare through three channels. First, it equalizes inventories: the matching session moves each trader toward the cross-sectional average inventory at the end of period 1, which lowers holding costs directly. Second, it provides information: the published aggregate Q improves the uninformed's estimate of the value of the asset at period 2, which lowers adverse selection and period 2 price impact, as shown in Section IIIE.1. Third, it distorts period 1 trade: anticipating the matching session, traders shade their period 1 demand schedules, which raises period 1 price impact and leaves traders with larger inventory imbalances whenever the matching session does not take place.

Table III reports welfare and price informativeness with and without the matching session. When there is a matching session, the conditional variance of the period 2 value for the uninformed falls from 0.514 to 0.110, reflecting the substantial information that Q adds

Table IIIWelfare and Price Informativeness
We report welfare by type and in aggregate, on each branch and combined, with a probabilistic matching session, $\theta = 0.1$, and without a matching session, $\theta = 0$. The row labeled "combined" is $(1-\theta)\,W^{\text{no}} + \theta\,W^{\text{SD}}$. $\text{Var}(v_2 \mid \mathcal{F}_U^{(2)})$ is the conditional variance of the period 2 value for an uninformed trader given their period 2 information set, and $\text{Var}(v_2 \mid p_2)$ is the conditional variance of the period 2 value given the period 2 price. The table uses the baseline parameterization.

	Matching ($\theta = 0.1$)	None ($\theta = 0$)	Difference
$W^{I,\text{SD}}$	-0.0143	-0.0299	$+0.0155$
$W^{U,\text{SD}}$	-0.0901	-0.1151	$+0.0250$
$W^{I,\text{no}}$	-0.0338	-0.0299	-0.0039
$W^{U,\text{no}}$	-0.1146	-0.1151	$+0.0006$
W^{I} (combined)	-0.0318	-0.0299	-0.0019
W^{U} (combined)	-0.1121	-0.1151	$+0.0030$
W^{SD}	-0.05223	-0.07250	$+0.02027$
W^{no}	-0.07416	-0.07250	-0.00165
W (combined)	-0.07196	-0.07250	$+0.00054$
$\text{Var}(v_2 \mid \mathcal{F}_U^{(2)})^{\text{SD}}$	0.1095	0.5138	-0.4042
$\text{Var}(v_2 \mid \mathcal{F}_U^{(2)})^{\text{no}}$	0.5034	0.5138	-0.0104
$\text{Var}(v_2 \mid p_2)^{\text{SD}}$	0.2693	0.5465	-0.2772
$\text{Var}(v_2 \mid p_2)^{\text{no}}$	0.5403	0.5465	-0.0062

to their period 2 information set. The period 2 price also becomes substantially more informative. Conditional on the matching session taking place, welfare is much higher for both types, and the uninformed capture a larger fraction of the aggregate gains. The large welfare gain on conditional on a matching session occurring is realized with probability $\theta = 0.1$. When there is no matching session, which occurs with probability $1-\theta = 0.9$, period 1 trade is distorted by the anticipation of the matching session, and welfare is lower than without a matching session. With forty traders, the distortion is small, and the loss when there is no matching session does not outweigh the gain on when a matching session occurs. Welfare differs across the two types. Combined across branches, the welfare of the uninformed increases by 0.003, while the welfare of the informed decreases by 0.002. Anticipating the matching session, the informed trade slightly more aggressively on their signals at period 1, so the period 1 price is more informative on both branches. This lowers the trading profits

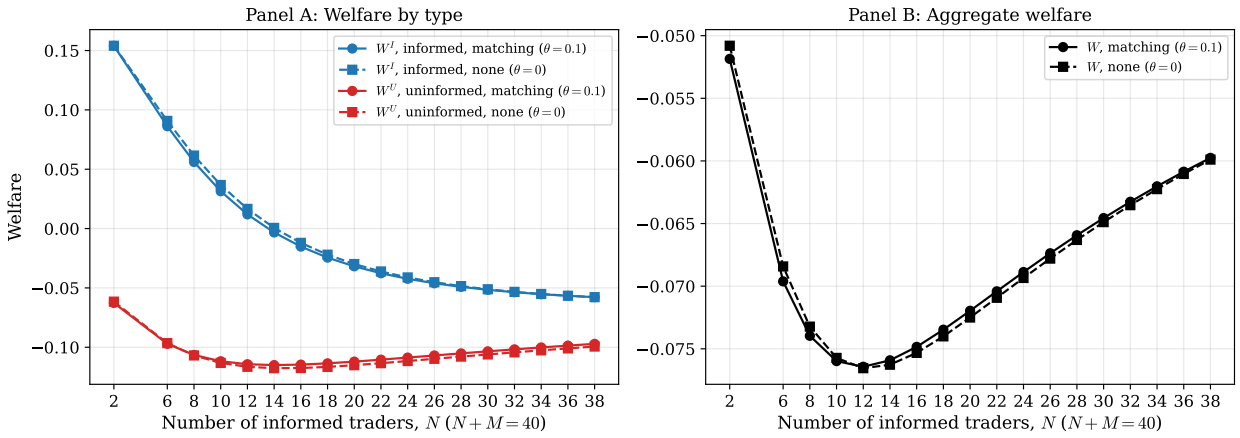

Figure 4. Welfare as a Function of the Composition of Traders
We plot welfare by type (Panel A) and aggregate welfare (Panel B) as a function of the number of informed traders, N, holding $N + M = 40$ fixed. Solid lines report the combined welfare with a probabilistic matching session, $\theta = 0.1$, and dashed lines report the benchmark without a matching session, $\theta = 0$. All other parameters are at the baseline parameterization.

of the informed and raises the welfare of the uninformed even when the matching session does not take place. The informed are better off than the uninformed with and without the matching session, even though they bear higher holding costs, because they earn trading profits at the expense of the uninformed.

B. Comparative Statics

We now study how the effects of the matching session vary with the parameters of the model. Figure 4 and Panel A of Table IV vary the composition of informed and uninformed traders, holding the total number of traders fixed. The matching session lowers period 2 price impact and the conditional variance of the period 2 value for the uninformed when there is a matching session. The net welfare effect is negative with ten or fewer informed traders and positive with twelve or more, and it is largest when the market is evenly split. With few informed traders, the matching session is most informative, but the increase in period 1 price impact from the anticipation of the matching session is also largest, and the loss when there is no matching session outweighs any gains. With more informed traders, period 1 price impact barely increases, and the gain when there is no matching session

dominates. Aggregate welfare is lowest when about a third of traders are informed, both with and without a matching session. Welfare differs across the two types. The welfare of the informed falls as the number of informed traders increases, as they compete away their trading profits, while the welfare of the uninformed is lowest when about fourteen traders are informed. The matching session increases the welfare of the uninformed whenever at least eight traders are informed, and it lowers the welfare of the informed for nearly every composition, with the largest loss when about eight traders are informed.

Panel B varies the probability of a matching session. The welfare effect is positive for every value of θ. It increases in θ at first, but the gain flattens and peaks at $\theta = 0.25$, since the increase in period 1 price impact grows faster than linearly in θ. Period 1 price impact increases in θ, since the more likely the matching session, the more traders shade their period 1 demand schedules in anticipation.

Panel C varies the variance of the innovation in the value of the asset between periods. As the innovation becomes larger, the information gap between the informed and uninformed grows from the matching session to date 2 trade. The reduction in the conditional variance of the period 2 value shrinks. The welfare gain from the matching session reduces as a function of the variance of the innovations.

Panel D varies the probability that the asset survives to period 2. The welfare gain increases in δ. As period 2 receives more weight, the information in Q becomes more important as the liquidation probability before it can be exploited decreases. The incentive to postpone trade to the matching session also grows, but with many traders the increase in period 1 price impact remains small.

Panel E varies the variance of inventory shocks. The welfare gain is largest at the benchmark parameterization and smaller for both smaller and larger inventory shocks. With larger inventory shocks, Q is more informative, and the learning component rises at first. The equalization component, however, is negative, and it becomes more negative as inventory shocks grow. The transfer is more valuable when traders have larger imbalances to equalize, but there is also more inventory adjustment to postpone, and the cost of this delay, which is paid whenever the matching session does not take place, grows by more. For large enough inventory shocks, this outweighs the additional learning, and the net gain shrinks.

Finally, Panel F varies the noise in the uninformed's signals. As the uninformed's signals become noisier, the matching session is more informative, and the reduction in their condi-

tional variance from observing Q grows. The welfare gain increases as well. The matching session improves welfare in every parameterization in Table IV, except when few traders are informed, where the increase in period 1 price impact from the anticipation of the matching session is largest.

V. Conclusion

This paper studies a dynamic model of strategic traders with heterogeneous information, in which a matching session, where further trade can take place at the fixed prior market-clearing price, may follow the first exchange session. The resulting cross-period price and demand impacts imply that the model is substantially more challenging to solve than stationary models of strategic trade. To address these challenges, we employ the methods of Rostek et al. (2025) in order to tractably compute the cross-period demand and price impacts.

The results show that matching sessions play an important informational role. The matching session demands weighs inventories and the value of the asset differently from the exchange price. It therefore provides information beyond the price, reducing adverse selection and price impact in the subsequent exchange session and improving price informativeness. However, the anticipation of the matching session leads traders to postpone trade from the preceding exchange session. When there are enough traders, this distortion is small, and the matching session increases welfare.

Table IVComparative Statics

We report the change in each variable from the benchmark without a matching session to the equilibrium with a probabilistic matching session, varying one parameter at a time. $\Delta\bar{\lambda}_1$ is the change in the population-weighted period 1 price impact, and $\Delta\bar{\lambda}_2^{\text{SD}}$ is the change in the population-weighted period 2 price impact on the matching session branch. $\Delta\text{Var}(v_2 \mid \mathcal{F}_U^{(2)})^{\text{SD}}$ and $\Delta\text{Var}(v_2 \mid p_2)^{\text{SD}}$ are the changes in the conditional variances of the period 2 value on the matching session branch. ΔW is the change in combined aggregate welfare. The number of informed traders is varied holding $N + M = 40$ fixed. Dashes indicate parameterizations for which we find no equilibrium with a matching session. All other parameters are at the baseline parameterization.

Value	$\Delta\bar{\lambda}_1$	$\Delta\bar{\lambda}_2^{\text{SD}}$	$\Delta\text{Var}(v_2 \mid \mathcal{F}_U^{(2)})^{\text{SD}}$	$\Delta\text{Var}(v_2 \mid p_2)^{\text{SD}}$	ΔW
Panel A: Number of informed traders, N					
8	+0.0047	−0.0087	−0.755	−0.614	−0.00073
16	+0.0010	−0.0108	−0.493	−0.365	+0.00048
24	+0.0002	−0.0094	−0.337	−0.211	+0.00048
32	+0.0002	−0.0069	−0.247	−0.122	+0.00027
Panel B: Probability of a matching session, θ					
0.025	+0.0001	−0.0103	−0.404	−0.277	+0.00015
0.1	+0.0004	−0.0103	−0.404	−0.277	+0.00054
0.2	+0.0012	−0.0103	−0.404	−0.277	+0.00086
0.3	+0.0026	−0.0103	−0.404	−0.277	+0.00085
Panel C: Variance of the value innovation, σ_η^2					
0.05	+0.0004	−0.0099	−0.418	−0.292	+0.00059
0.2	+0.0004	−0.0092	−0.378	−0.254	+0.00042
0.5	+0.0005	−0.0048	−0.328	−0.218	+0.00018
Panel D: Probability the asset survives to period 2, δ					
0.5	+0.0004	−0.0103	−0.404	−0.277	+0.00054
0.7	+0.0009	−0.0083	−0.392	−0.270	+0.00081
Panel E: Inventory shock variance, σ_h^2					
0.5	+0.0005	−0.0163	−0.278	−0.212	+0.00039
1	+0.0004	−0.0103	−0.404	−0.277	+0.00054
4	+0.0006	−0.0033	−0.694	−0.311	+0.00010
Panel F: Noise in uninformed signals, $\sigma_{\epsilon,U}^2$					
0.5	+0.0003	−0.0068	−0.080	−0.095	+0.00018
4	+0.0004	−0.0109	−0.274	−0.209	+0.00043
16	+0.0004	−0.0108	−0.361	−0.255	+0.00051
∞	+0.0004	−0.0103	−0.404	−0.277	+0.00054

VI. Works Cited

Antill, Samuel, and Darrell Duffie, 2020, Augmenting Markets with Mechanisms, *The Review of Economic Studies* 88, 1665–1719.

Blonien, Patrick, 2026, Size Discovery in Slow Markets, *SSRN* .

Blonien, Patrick, and Alexander Ober, 2026, Is 24/7 Trading Better?, *SSRN* .

Blume, Lawrence, David Easley, and Maureen O'Hara, 1994, Market statistics and technical analysis: The role of volume, *Journal of Finance* 49, 153–181.

Buti, Sabrina, Barbara Rindi, and Ingrid M. Werner, 2017, Dark pool trading strategies, market quality, and welfare, *Journal of Financial Economics* 124, 244–265.

Chen, Daniel, 2022, Optimal Design of a Financial Exchange, *SSRN* .

Chen, Daniel, and Darrell Duffie, 2021, Market fragmentation, *American Economic Review* 111, 2247–2274.

Comerton-Forde, Carole, and Tālis J. Putniņš, 2015, Dark trading and price discovery, *Journal of Financial Economics* 118, 70–92.

Degryse, Hans, Frank de Jong, and Vincent van Kervel, 2014, The Impact of Dark Trading and Visible Fragmentation on Market Quality, *Review of Finance* 19, 1587–1622.

Du, Songzi, and Haoxiang Zhu, 2017, What is the Optimal Trading Frequency in Financial Markets?, *The Review of Economic Studies* 84, 1606–1651.

Duffie, Darrell, and Haoxiang Zhu, 2017, Size Discovery, *The Review of Financial Studies* 30, 1095–1150.

Fleming, Michael J, and Giang Nguyen, 2018, Price and Size Discovery in Financial Markets: Evidence from the U.S. Treasury Securities Market, *The Review of Asset Pricing Studies* 9, 256–295.

Ganguli, Jayant Vivek, and Liyan Yang, 2009, Complementarities, multiplicity, and supply information, *Journal of the European Economic Association* 7, 90–115.

Grossman, Sanford J., 1976, On the efficiency of competitive stock markets where trades have diverse information, *Journal of Finance* 31, 573–585.

Hatheway, Frank, Amy Kwan, and Hui Zheng, 2017, An empirical analysis of market segmentation on U.S. equity markets, *Journal of Financial and Quantitative Analysis* 52, 2399–2427.

Hayek, Friedrich A., 1945, The use of knowledge in society, *American Economic Review* 35, 519–530.

Hendershott, Terrence, and Haim Mendelson, 2000, Crossing networks and dealer markets: Competition and performance, *Journal of Finance* 55, 2071–2115.

Klemperer, Paul D., and Margaret A. Meyer, 1989, Supply function equilibria in oligopoly under uncertainty, *Econometrica* 57, 1243–1277.

Kyle, Albert S., 1989, Informed Speculation with Imperfect Competition, *The Review of Economic Studies* 56, 317–355.

Manzano, Carolina, and Xavier Vives, 2011, Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity, *Journal of Mathematical Economics* 47, 346–369.

Menkveld, Albert J., Bart Zhou Yueshen, and Haoxiang Zhu, 2017, Shades of darkness: A pecking order of trading venues, *Journal of Financial Economics* 124, 503–534.

Rostek, Marzena, and Marek Weretka, 2012, Price inference in small markets, *Econometrica* 80, 687–711.

Rostek, Marzena J, Ji Hee Yoon, and Xian Wu, 2025, Dynamic imperfectly competitive markets: A non-recursive approach Working Paper.

Sannikov, Yuliy, and Andrzej Skrzypacz, 2016, Dynamic Trading: Price Inertia and Front-Running, *SSRN* .

Schneider, Jan, 2009, A rational expectations equilibrium with informative trading volume, *Journal of Finance* 64, 2783–2805.

Securities and Exchange Commission, 2010, Concept release on equity market structure, Technical Report Release No. 34-61358; File No. S7-02-10, Securities and Exchange Commission, 75 Fed. Reg. 3594.

Vayanos, Dimitri, 1999, Strategic Trading and Welfare in a Dynamic Market, *The Review of Economic Studies* 66, 219–254.

Vives, Xavier, 2011, Strategic supply function competition with private information, *Econometrica* 79, 1919–1966.

Zhu, Haoxiang, 2013, Do Dark Pools Harm Price Discovery?, *The Review of Financial Studies* 27, 747–789.

Appendix

A. FOC Derivations and Matching Conditions

This appendix derives the equilibrium FOCs and matching conditions that pin down the strategy coefficients of the linear strategies (70), (51), and (35). We first state the trader's problem. The derivation then proceeds backward through the timeline: period 2 (terminal exchange), matching session stage 1B, then period 1 (initial exchange). We describe the equilibrium assuming there is a matching session for certain. The probabilistic matching session is an extension.

A. The Trader's Problem

Throughout, v_t denotes the common value of the asset at period t, which follows a random walk, p_t the period t exchange price, D_t^i trader i's period t exchange demand, S_t^i trader i's private signal about v_t, and h_t^i trader i's period t inventory shock. At stage 1B, trader i submits a matching session demand μ_i, the aggregate demand $Q = \sum_j \mu_j$ is published, and trader i receives the matching session transfer $D_{SD}^i = \mu_i - Q/(N+M)$, their demand net of the average demand, at the period 1 price p_1. Inventories evolve as

$$\hat{z}_1^i = h_1^i + D_1^i, \qquad z_1^i = \hat{z}_1^i + D_{SD}^i,$$
$$\hat{z}_2^i = z_1^i + h_2^i, \qquad z_2^i = \hat{z}_2^i + D_2^i,$$

so that $\hat{z}_t^i$ is the inventory immediately before, and z_t^i the inventory after, the last trading session of period t. There are N informed traders and M uninformed traders. When a formula applies to either type, n_τ denotes the number of traders of type $\tau \in \{I, U\}$, so that $n_I = N$ and $n_U = M$, and $-\tau$ denotes the type other than τ.

Trader i of type τ maximizes

$$\mathbb{E}\Big[- p_1\big(D_1^i + D_{SD}^i\big) + (1-\delta)\Big(v_1\big(\hat{z}_1^i + D_1^i + D_{SD}^i\big) - \frac{\gamma}{2}\big(\hat{z}_1^i + D_1^i + D_{SD}^i\big)^2\Big) + \delta\Big(- p_2 D_2^i + v_2\big(\hat{z}_2^i + D_2^i\big) - \frac{\gamma}{2}\big(\hat{z}_2^i + D_2^i\big)^2\Big)\Big]. \quad (26)$$

Holding inventory z costs $\frac{\gamma}{2}z^2$, so the marginal holding cost is γz. Denote the ex-post realized continuation value as

$$V_{i,2}(\hat{z}_2^i, D_2^i, p_2) = -p_2 D_2^i + v_2\big(\hat{z}_2^i + D_2^i\big) - \frac{\gamma}{2}\big(\hat{z}_2^i + D_2^i\big)^2.$$

Therefore, before period 2, traders maximize

$$\mathbb{E}\Big[-p_1\big(D_1^i + D_{SD}^i\big) + (1-\delta)\Big(v_1 z_1^i - \frac{\gamma}{2}\big(z_1^i\big)^2\Big) + \delta V_{i,2}(\hat{z}_2^i, D_2^i, p_2)\Big].$$

Trader i of type τ has information set $\mathcal{F}_i^{(1),\tau}$ at period 1, $\mathcal{F}_i^{(\mathrm{SD}),\tau}$ at stage 1B, and $\mathcal{F}_i^{(2),\tau}$ at period 2, defined in (37) and (27). Because traders submit demand schedules, the period t information set includes the clearing price p_t.

Define the own-period price impacts of trader i of type τ to be the constants $\lambda_1^\tau := \frac{\mathrm{d}p_1}{\mathrm{d}D_1^i}$ and $\lambda_2^\tau := \frac{\mathrm{d}p_2}{\mathrm{d}D_2^i}$, and the cross-period price impacts to be the constants $\lambda_{1,2}^\tau := \frac{\mathrm{d}p_2}{\mathrm{d}D_1^i}$ and $\lambda_{\mathrm{SD},2}^\tau := \frac{\mathrm{d}p_2}{\mathrm{d}\mu_i}$. These are determined in Appendix AE.4.

B. period 2 FOC

At period 2, trader i of type τ has information set

$$\mathcal{F}_i^{(2),\tau} = \big\{S_1^i,\ S_2^i,\ p_1,\ p_2,\ h_1^i,\ h_2^i,\ Q\big\}, \tag{27}$$

where $Q = \sum_j \mu_j$ is the aggregate demand in the matching session.

Conjecture that trader i of type τ has linear beliefs at period 2. The projection of v_2 onto $\mathcal{F}_i^{(2),\tau}$ is

$$\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] = c_{S_1,2}^\tau S_1^i + c_{S_2,2}^\tau S_2^i + c_{p_1,2}^\tau p_1 + c_{p_2,2}^\tau p_2 + c_{h_1,2}^\tau h_1^i + c_{h_2,2}^\tau h_2^i + c_{Q,2}^\tau Q, \tag{28}$$

a function of seven coefficients.

FOC and demand schedule. Differentiating (26) with respect to D_2^i and taking the conditional expectation given $\mathcal{F}_i^{(2),\tau}$, the common factor δ drops out and the FOC at period 2 is

$$\mathbb{E}\big[v_2 - \gamma\big(\hat{z}_2^i + D_2^i\big) \;\big|\; \mathcal{F}_i^{(2),\tau}\big] = p_2 + \lambda_2^\tau D_2^i. \tag{29}$$

The price-impact term $\lambda_2^\tau D_2^i$ captures the trader's own marginal effect on p_2 via market clearing. Solving (29) for D_2^i:

$$\implies D_2^i(p_2) = \chi_2^\tau\left(\frac{1}{\gamma}\big(\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] - p_2\big) - \hat{z}_2^i\right), \qquad \chi_2^\tau := \frac{\gamma}{\gamma + \lambda_2^\tau}. \tag{30}$$

The trader's period 2 demand has two components: a valuation-price gap ($\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] - p_2$), scaled by the marginal cost of holding more inventory, and an exit from their current inventory position ($-\hat{z}_2^i$). The term in the large parentheses is the demand a trader would submit in a perfectly competitive market. It is scaled by $\chi_2^\tau \in [0,1]$, which bid shades

demand due to imperfect competition.

Market clearing and own-period price impact. Aggregating $\sum_j D_2^j(p_2) = 0$, the period 2 clearing price is

$$p_2^* = \frac{\sum_\tau n_\tau \chi_2^\tau \left(\frac{1}{\gamma}\overline{V}_2^\tau - \overline{\hat{z}}_2^\tau\right)}{\sum_\tau n_\tau \chi_2^\tau \frac{1}{\gamma}(1 - c_{p_2,2}^\tau)}, \tag{31}$$

where $\overline{V}_2^\tau$ is the average across type-τ traders of their conditional valuation $\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] - c_{p_2,2}^\tau p_2$ (stripping out the indirect dependence on p_2 through the conditional expectation), and $\overline{\hat{z}}_2^\tau$ is the type-average inventory.

The own-period price impact is a type-specific fixed point, $\lambda_2^\tau \equiv \frac{\mathrm{d}p_2}{\mathrm{d}D_2^i} = -\left(\sum_{j\neq i} \frac{\partial D_2^j}{\partial p_2}\right)^{-1}$, which is

$$\lambda_2^I = \frac{\gamma}{(N-1)\chi_2^I(1 - c_{p_2,2}^I) + M\chi_2^U(1 - c_{p_2,2}^U)}, \tag{32}$$

$$\lambda_2^U = \frac{\gamma}{N\chi_2^I(1 - c_{p_2,2}^I) + (M-1)\chi_2^U(1 - c_{p_2,2}^U)}, \tag{33}$$

which, after substituting $\chi_2^\tau = \gamma(\gamma + \lambda_2^\tau)^{-1}$, is a non-linear system in $(\lambda_2^I, \lambda_2^U)$.

Demand slope with price feedback. Because $\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}]$ depends on p_2 through $c_{p_2,2}^\tau$, the trader's effective demand slope on p_2 is

$$b_2^\tau := \left|\frac{\partial D_2^i}{\partial p_2}\right| = \chi_2^\tau \frac{1 - c_{p_2,2}^\tau}{\gamma}. \tag{34}$$

The numerator $1 - c_{p_2,2}^\tau$ measures the trader's net sensitivity to a unit increase in p_2: the cost of buying rises by 1, but the conditional expectation of v_2 rises by $c_{p_2,2}^\tau$ (the price carries information).

Strategy matching conditions. The period 2 strategy of trader i of type τ is the linear demand

$$D_2^i = \sum_\xi a_{2,\xi}^\tau \, \xi = a_{2,S_1}^\tau S_1^i + a_{2,S_2}^\tau S_2^i + a_{2,h_1}^\tau h_1^i + a_{2,h_2}^\tau h_2^i + a_{2,p_1}^\tau p_1 + a_{2,p_2}^\tau p_2 + a_{2,Q}^\tau Q, \tag{35}$$

where ξ ranges over the $\mathcal{F}^{(2)}$-basis directions $\{S_1, S_2, h_1, h_2, p_1, p_2, Q\}$. Substituting into (29) and matching coefficients yields

$$a^\tau_{2,\xi} = \chi^\tau_2 \left(\frac{1}{\gamma} \big(c^\tau_{\xi,2} - \mathbf{1}\{\xi = p_2\} \big) - z^\tau_\xi \right), \tag{36}$$

where z^τ_ξ is the loading of $\hat{z}^i_2$ on direction ξ in the $\mathcal{F}^{(2)}$ basis.

The z^τ_ξ loadings depend on the design. Letting $\hat{z}^i_2 = z^i_1 + h^i_2$, we have:

No Matching Session. Without matching session, $z^i_1 = h^i_1 + D^i_1$, so

$$\begin{aligned} z^\tau_{S_1} &= a^\tau_{1,S}, & z^\tau_{S_2} &= 0, \\ z^\tau_{h_1} &= 1 + a^\tau_{1,h}, & z^\tau_{h_2} &= 1, \\ z^\tau_{p_1} &= a^\tau_{1,p}, & z^\tau_{p_2} &= 0, \\ z^\tau_Q &= 0. \end{aligned}$$

Matching Session. With a matching session, $z^i_1 = h^i_1 + D^i_1 + \mu_i - Q/(N+M)$. Let $\mu_i = b^\tau_z \hat{z}^i_1 + b^\tau_S S^i_1 + b^\tau_h h^i_1$. Using z^i_1's primitive representation, the coefficient of z^i_1 on direction ξ in the $\mathcal{F}^{(2)}$ basis yields

$$\begin{aligned} z^\tau_{S_1} &= a^\tau_{1,S}\,(1 + b^\tau_z) + b^\tau_S, & z^\tau_{S_2} &= 0, \\ z^\tau_{h_1} &= (1 + a^\tau_{1,h})(1 + b^\tau_z) + b^\tau_h, & z^\tau_{h_2} &= 1, \\ z^\tau_{p_1} &= a^\tau_{1,p}(1 + b^\tau_z), & z^\tau_{p_2} &= 0, \\ z^\tau_Q &= -1/(N+M). \end{aligned}$$

Equations for intuition. Define the period 2 weight of type τ as

$$\omega^\tau_2 := \frac{\frac{n_\tau}{\gamma + \lambda^\tau_2}}{\frac{M}{\gamma + \lambda^U_2} + \frac{N}{\gamma + \lambda^I_2}},$$

and let $\overline{\hat{z}}^\tau_2$ denote the type-average inventory, as in (31). Aggregating (30) across traders, the period 2 clearing price can equivalently be written (implicitly, since the conditional expectations depend on p_2) as

$$p^*_2 = \sum_{\tau \in \{I,U\}} \omega^\tau_2 \left(\frac{1}{n_\tau} \sum_{j \in \tau} \mathbb{E}[v_2 \mid \mathcal{F}^{(2),\tau}_j] - \gamma\, \overline{\hat{z}}^\tau_2 \right),$$

where $j \in \tau$ denotes the traders of type τ. The equilibrium demand of trader i of type τ is then

$$D_2^i(p_2^*) = \chi_2^\tau \left[\frac{1}{\gamma} \left(\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] - \sum_{\tau' \in \{I,U\}} \omega_2^{\tau'} \frac{1}{n_{\tau'}} \sum_{j \in \tau'} \mathbb{E}[v_2 \mid \mathcal{F}_j^{(2),\tau'}] \right) - \left(\hat{z}_2^i - \sum_{\tau' \in \{I,U\}} \omega_2^{\tau'} \overline{\hat{z}}_2^{\tau'} \right) \right].$$

The post-trade inventory is then

$$\hat{z}_2^i + D_2^i(p_2^*) = \frac{1}{\gamma + \lambda_2^\tau} \left(\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] - \sum_{\tau' \in \{I,U\}} \omega_2^{\tau'} \frac{1}{n_{\tau'}} \sum_{j \in \tau'} \mathbb{E}[v_2 \mid \mathcal{F}_j^{(2),\tau'}] \right) + \frac{\lambda_2^\tau}{\gamma + \lambda_2^\tau} \hat{z}_2^i + \frac{\gamma}{\gamma + \lambda_2^\tau} \sum_{\tau' \in \{I,U\}} \omega_2^{\tau'} \overline{\hat{z}}_2^{\tau'}.$$

C. Matching Session FOC (Stage 1B)

At stage 1B, trader i submits a matching session demand μ_i given information set

$$\mathcal{F}_i^{(\mathrm{SD}),\tau} = \mathcal{F}_i^{(1),\tau} = \{S_1^i,\ h_1^i,\ p_1\}, \tag{37}$$

where p_1 is the just-cleared stage-1A price. Then simultaneously, the sum $Q = \sum_j \mu_j$ is published and the matching session transfers

$$D_{SD}^i = \mu_i - Q/(N+M), \tag{38}$$

so that each trader receives their demand net of the average demand. The transfer is settled at the stage-1A price p_1, so that the post-matching session inventory is

$$z_1^i = \hat{z}_1^i + D_{SD}^i = h_1^i + D_1^i + \mu_i - Q/(N+M). \tag{39}$$

By construction, $\sum_j D_{SD}^j = 0$, so the matching session mechanism is budget-balanced.

Marginal effects of μ_i. Differentiating the various relevant equilibrium objects with respect to μ_i:

$$\frac{\mathrm{d}z_1^i}{\mathrm{d}\mu_i} = \frac{N+M-1}{N+M}, \tag{40}$$

$$\frac{\mathrm{d}\hat{z}_2^i}{\mathrm{d}\mu_i} = \frac{\mathrm{d}z_1^i}{\mathrm{d}\mu_i} = \frac{N+M-1}{N+M}, \tag{41}$$

$$\frac{\mathrm{d}D_{SD}^i}{\mathrm{d}\mu_i} = \frac{N+M-1}{N+M}, \tag{42}$$

$$\frac{\mathrm{d}p_2}{\mathrm{d}\mu_i} = \lambda_{\mathrm{SD},2}^\tau, \tag{43}$$

$$\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i} = -\chi_2^\tau\Big(\frac{N+M-1}{N+M} + \frac{\lambda_{\mathrm{SD},2}^\tau}{\gamma}\Big). \tag{44}$$

The result (44) follows from implicit differentiation of the period 2 FOC (Appendix AE.1).

FOC. At stage 1B, the period 1 price p_1 has already cleared, so the matching session demand has no impact on p_1. Traders choose μ_i to maximize

$$\mathbb{E}\Big[- p_1 D_{SD}^i + (1-\delta)\Big(v_1(\hat{z}_1^i + D_{SD}^i) - \frac{\gamma}{2}\big(\hat{z}_1^i + D_{SD}^i\big)^2\Big) + \delta V_{i,2}(\hat{z}_2^i, D_2^i, p_2) \;\Big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\Big],$$

hence their FOC is

$$\mathbb{E}\Big[(1-\delta)\,\frac{N+M-1}{N+M}\,\big(v_1 - \gamma(\hat{z}_1^i + D_{SD}^i)\big) + \delta\,\frac{\mathrm{d}V_{i,2}}{\mathrm{d}\mu_i} \;\Big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\Big] = \frac{N+M-1}{N+M}\,p_1. \tag{45}$$

Using the period 2 FOC, $\mathbb{E}[v_2 - \gamma z_2^i - p_2 \mid \mathcal{F}_i^{(2),\tau}] = \lambda_2^\tau D_2^i$, the marginal continuation value is

$$\begin{aligned}
\frac{\mathrm{d}V_{i,2}}{\mathrm{d}\mu_i} &= -\frac{\mathrm{d}p_2}{\mathrm{d}\mu_i}D_2^i - p_2\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i} + v_2\Big(\frac{N+M-1}{N+M} + \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}\Big) - \gamma\big(\hat{z}_2^i + D_2^i\big)\Big(\frac{N+M-1}{N+M} + \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}\Big) \\
&= \big(v_2 - \gamma z_2^i\big)\Big(\frac{N+M-1}{N+M} + \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}\Big) - p_2\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i} - \lambda_{\mathrm{SD},2}^\tau D_2^i \\
&= \big(v_2 - \gamma z_2^i - p_2\big)\left(\frac{N+M-1}{N+M} + \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i} - \frac{\lambda_{\mathrm{SD},2}^\tau}{\lambda_2^\tau}\right) + \frac{N+M-1}{N+M}\,p_2 \\
&= \big(v_2 - \gamma z_1^i - p_2\big)(1-\chi_2^\tau)\left(\frac{N+M-1}{N+M} + \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i} - \frac{\lambda_{\mathrm{SD},2}^\tau}{\lambda_2^\tau}\right) + \frac{N+M-1}{N+M}\,p_2.
\end{aligned}$$

By the law of iterated expectations, we've implicitly eliminated mean 0 terms: the third equality uses the period 2 FOC, and the fourth uses the period 2 demand (30) together with $\hat{z}_2^i = z_1^i + h_2^i$ and $\mathbb{E}[h_2^i \mid \mathcal{F}_i^{(\mathrm{SD}),\tau}] = 0$. Note, therefore, that the FOC at stage 1B actually implies

$$\begin{aligned}
\frac{N+M-1}{N+M}\,p_1 = (1-\delta)\,\frac{N+M-1}{N+M}\,\mathbb{E}\big[v_1 - \gamma(\hat{z}_1^i + D_{SD}^i) \;\big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\big] \\
+ \delta\,(1-\chi_2^\tau)\Big(\frac{N+M-1}{N+M} + \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i} - \frac{\lambda_{\mathrm{SD},2}^\tau}{\lambda_2^\tau}\Big)\,\mathbb{E}\big[v_2 - \gamma(\hat{z}_1^i + D_{SD}^i) - p_2 \;\big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\big] \\
+ \delta\,\frac{N+M-1}{N+M}\,\mathbb{E}\big[p_2 \;\big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\big].
\end{aligned}$$

Using the random walk identity $\mathbb{E}[v_2 \mid \mathcal{F}_i^{(\mathrm{SD}),\tau}] = \mathbb{E}[v_1 \mid \mathcal{F}_i^{(\mathrm{SD}),\tau}]$ and combining terms, the FOC can be rewritten as

$$\mathbb{E}\big[v_1 - \gamma(\hat{z}_1^i + D_{SD}^i) \;\big|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\big] = \Omega_{\mathrm{SD}}^\tau\,p_1 + (1-\Omega_{\mathrm{SD}}^\tau)\,\mathbb{E}[p_2 \mid \mathcal{F}_i^{(\mathrm{SD}),\tau}], \tag{46}$$

where

$$\Omega^{\tau}_{\mathrm{SD}} = \frac{1}{1-\delta\big[1-(1-\chi^{\tau}_2)^2\big]-\delta\,(1-\chi^{\tau}_2)(2-\chi^{\tau}_2)\,\dfrac{(N+M)\,\lambda^{\tau}_{\mathrm{SD},2}}{(N+M-1)\,\lambda^{\tau}_2}}. \tag{47}$$

Solving for D^i_{SD}, we get

$$\implies \mathbb{E}\big[D^i_{SD}\;\big|\;\mathcal{F}^{(\mathrm{SD}),\tau}_i\big] = \frac{1}{\gamma}\Big(\mathbb{E}[v_1-p_2\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]-\Omega^{\tau}_{\mathrm{SD}}\big(p_1-\mathbb{E}[p_2\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]\big)\Big)-\hat{z}^i_1. \tag{48}$$

This expected allocation can be rewritten as the expected competitive allocation plus an additional term due to the imperfect competition,

$$\begin{aligned}\mathbb{E}\big[D^i_{SD}\;\big|\;\mathcal{F}^{(\mathrm{SD}),\tau}_i\big] = {}& \underbrace{\frac{1}{\gamma}\left(\mathbb{E}[v_1\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]-\frac{p_1-\delta\,\mathbb{E}[p_2\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]}{1-\delta}\right)-\hat{z}^i_1}_{\text{competitive demand}}\\ &+\underbrace{\frac{1}{\gamma}\Big(\frac{1}{1-\delta}-\Omega^{\tau}_{\mathrm{SD}}\Big)\big(p_1-\mathbb{E}[p_2\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]\big)}_{\text{imperfect-competition correction}}.\end{aligned} \tag{49}$$

The quantity submitted in the matching session is then

$$\mu_i = \frac{1}{\gamma}\Big(\mathbb{E}[v_1\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]-\Omega^{\tau}_{\mathrm{SD}}p_1-(1-\Omega^{\tau}_{\mathrm{SD}})\,\mathbb{E}[p_2\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]\Big)-\hat{z}^i_1+\frac{1}{N+M}\,\mathbb{E}\big[Q\;\big|\;\mathcal{F}^{(\mathrm{SD}),\tau}_i\big]. \tag{50}$$

Workup matching conditions. Conjecture the linear matching session demand

$$\mu_i = b^{\tau}_z\,\hat{z}^i_1 + b^{\tau}_S\,S^i_1 + b^{\tau}_h\,h^i_1. \tag{51}$$

The pre-matching session inventory $\hat{z}^i_1$ has $\mathcal{F}^{(\mathrm{SD})}$-basis loadings

$$z^{\tau}_{S_1} = a^{\tau}_{1,S},\quad z^{\tau}_{h_1} = 1+a^{\tau}_{1,h},\quad z^{\tau}_{p_1} = a^{\tau}_{1,p}. \tag{52}$$

Define the FOC-implied numerator ν^{τ}_{ξ} on each direction $\xi\in\{S,h,p_1\}$, that is, the loading on ξ of the term in parentheses in (48) after applying the random walk identity:

$$\nu^{\tau}_S = c^{\tau}_{S,1}-(1-\Omega^{\tau}_{\mathrm{SD}})\rho^{\tau}_{S,2}, \tag{53}$$

$$\nu^{\tau}_h = c^{\tau}_{h,1}-(1-\Omega^{\tau}_{\mathrm{SD}})\rho^{\tau}_{h,2}, \tag{54}$$

$$\nu^{\tau}_{p_1} = c^{\tau}_{p_1,1}-\Omega^{\tau}_{\mathrm{SD}}-(1-\Omega^{\tau}_{\mathrm{SD}})\rho^{\tau}_{p_1,2}. \tag{55}$$

The c projections are loadings of $\mathbb{E}[v_1\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]$, and the ρ projections are loadings of $\mathbb{E}[p_2\mid\mathcal{F}^{(\mathrm{SD}),\tau}_i]$.

Since $D^i_{SD}=\mu_i-Q/(N+M)$, the left-hand side of (48) is $\frac{N+M-1}{N+M}\,\mu_i-\mathbb{E}\big[\frac{1}{N+M}\sum_{j\neq i}\mu_j\;\big|$

$\mathcal{F}_i^{(\mathrm{SD}),\tau}\big]$. The matching condition on each direction $\xi \in \{S, h, p_1\}$ of $\mathcal{F}_i^{(\mathrm{SD}),\tau}$ is

$$\frac{N+M-1}{N+M}\big(b_z^\tau z_\xi^\tau + b_\xi^\tau \mathbf{1}\{\xi \neq p_1\}\big) - \pi_\xi^\tau = \frac{\nu_\xi^\tau}{\gamma} - z_\xi^\tau, \tag{56}$$

where π_ξ^τ is the rival-aggregate projection $\pi_\xi^\tau =$ loading of $\frac{1}{N+M}\sum_{j\neq i}\mu_j^*$ on direction ξ. The three matching equations (56) pin down the three coefficients (b_z, b_S, b_h):

$$\frac{N+M-1}{N+M}\, b_z^\tau\, a_{1,p}^\tau - \pi_{p_1}^\tau = \frac{\nu_{p_1}^\tau}{\gamma} - a_{1,p}^\tau, \tag{57}$$

$$b_\xi^\tau = \frac{N+M}{N+M-1}\Big(\frac{\nu_\xi^\tau}{\gamma} - z_\xi^\tau + \pi_\xi^\tau\Big) - b_z^\tau\, z_\xi^\tau, \qquad \xi \in \{S, h\}. \tag{58}$$

The b_z coefficient is determined by matching p_1.

D. period 1 FOC

At period 1, trader i submits a demand schedule $D_1^i(p)$ prior to observing p_1. The realized $D_1^i = D_1^i(p_1)$ becomes a function of $\mathcal{F}_i^{(1),\tau} = \{S_1^i, h_1^i, p_1\}$ after the stage-1A market clears.

FOC. Both D_1^i and D_{SD}^i are settled at the period 1 price. At period 1, traders maximize

$$\mathbb{E}\Big[-p_1\big(D_1^i + D_{SD}^i\big) + (1-\delta)\Big(v_1 z_1^i - \frac{\gamma}{2}\big(z_1^i\big)^2\Big) + \delta V_{i,2}(\hat{z}_2^i, D_2^i, p_2) \;\Big|\; \mathcal{F}_i^{(1),\tau}\Big],$$

hence their FOC is

$$\begin{aligned}\mathbb{E}\Big[(1-\delta)\big(v_1 - \gamma z_1^i\big)\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i}\Big) + \delta\,\frac{\mathrm{d}V_{i,2}}{\mathrm{d}D_1^i} \;\Big|\; \mathcal{F}_i^{(1),\tau}\Big] \\ = p_1\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i}\Big) + \lambda_1^\tau\,\mathbb{E}\big[D_1^i + D_{SD}^i \;\big|\; \mathcal{F}_i^{(1),\tau}\big].\end{aligned} \tag{59}$$

Going back to the continuation value,

$$\begin{aligned}\frac{\mathrm{d}V_{i,2}}{\mathrm{d}D_1^i} &= -\frac{\mathrm{d}p_2}{\mathrm{d}D_1^i}D_2^i - p_2\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} + \big(v_2 - \gamma(\hat{z}_2^i + D_2^i)\big)\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i} + \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}\Big) \\ &= \big(v_2 - \gamma z_2^i\big)\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i} + \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}\Big) - p_2\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} - \lambda_{1,2}^\tau D_2^i \\ &= \big(v_2 - p_2 - \gamma z_2^i\big)\left(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i} + \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} - \frac{\lambda_{1,2}^\tau}{\lambda_2^\tau}\right) + p_2\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i}\Big) \\ &= \big(v_2 - p_2 - \gamma z_1^i\big)(1-\chi_2^\tau)\left(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i} + \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} - \frac{\lambda_{1,2}^\tau}{\lambda_2^\tau}\right) + p_2\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i}\Big).\end{aligned}$$

As at stage 1B, by the law of iterated expectations we've implicitly eliminated mean 0 terms: the third equality uses the period 2 FOC, and the fourth uses the period 2 demand

(30) together with $\hat{z}_2^i = z_1^i + h_2^i$ and $\mathbb{E}[h_2^i \mid \mathcal{F}_i^{(1),\tau}] = 0$. Define

$$\kappa_v^\tau \equiv (1-\delta)\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i}\Big) + \delta(1-\chi_2^\tau)\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i} + \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} - \frac{\lambda_{1,2}^\tau}{\lambda_2^\tau}\Big). \tag{60}$$

Using the random walk identity $\mathbb{E}[v_2 \mid \mathcal{F}_i^{(1),\tau}] = \mathbb{E}[v_1 \mid \mathcal{F}_i^{(1),\tau}]$, $z_1^i = h_1^i + D_1^i + D_{SD}^i$, and collecting terms, the FOC at period 1 can be rewritten as

$$\mathbb{E}\big[v_1 - \gamma z_1^i \,\big|\, \mathcal{F}_i^{(1),\tau}\big] = \Omega_1^\tau\, p_1 + (1-\Omega_1^\tau)\,\mathbb{E}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}\big] + \frac{\lambda_1^\tau}{\kappa_v^\tau}\,\mathbb{E}\big[D_1^i + D_{SD}^i \,\big|\, \mathcal{F}_i^{(1),\tau}\big], \tag{61}$$

where

$$\Omega_1^\tau := \frac{1}{\kappa_v^\tau}\Big(1 + \frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i}\Big). \tag{62}$$

This has the same form as the period 2 FOC (29) and the stage 1B FOC (46): the expected marginal value of the period 1 position equals an effective price, a weighted combination of p_1 and the expected period 2 price with weight Ω_1^τ on p_1, plus the price impact on the total period 1 trade, since both D_1^i and D_{SD}^i settle at p_1. Ω_1^τ is the period 1 counterpart of $\Omega_{\mathrm{SD}}^\tau$ in (46).

No matching session. Without a matching session, $D_{SD}^i \equiv 0$, so $\frac{\mathrm{d}D_{SD}^i}{\mathrm{d}D_1^i} = 0$, $\Omega_1^\tau = 1/\kappa_v^\tau$, and $z_1^i = h_1^i + D_1^i$. Solving (61) for D_1^i:

$$\Longrightarrow\; D_1^i(p_1) = \chi_1^\tau\bigg(\frac{1}{\gamma}\Big(\mathbb{E}\big[v_1 \,\big|\, \mathcal{F}_i^{(1),\tau}\big] - \Omega_1^\tau\, p_1 - (1-\Omega_1^\tau)\,\mathbb{E}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}\big]\Big) - h_1^i\bigg), \tag{63}$$

where $\chi_1^\tau := \gamma\kappa_v^\tau/(\gamma\kappa_v^\tau + \lambda_1^\tau)$, and κ_v^τ, the price impacts, and the projection of p_2 are those of the no-matching-session branch. As in (30) and (48), the demand is the competitive demand, a net benefit scaled by the marginal holding cost minus the current inventory, bid shaded by $\chi_1^\tau \in [0,1]$. Without price impact, $\kappa_v^\tau = 1-\delta$, $\chi_1^\tau = 1$, and the effective price is $\big(p_1 - \delta\mathbb{E}[p_2]\big)/(1-\delta)$, the cost of a unit bought at p_1 and resold at p_2 if the asset does not liquidate.

Matching session. With a matching session, the stage 1B FOC (46) gives the left-hand side of (61), and (48) gives $h_1^i + D_1^i + \mathbb{E}[D_{SD}^i \mid \mathcal{F}_i^{(1),\tau}] = \frac{1}{\gamma}\big(\mathbb{E}[v_1 \mid \mathcal{F}_i^{(1),\tau}] - \Omega_{\mathrm{SD}}^\tau p_1 - (1-\Omega_{\mathrm{SD}}^\tau)\mathbb{E}[p_2 \mid \mathcal{F}_i^{(1),\tau}]\big)$. Substituting both, D_1^i drops out and the FOC becomes

$$0 = \kappa_v^\tau\big(\Omega_{\mathrm{SD}}^\tau - \Omega_1^\tau\big)\Big(p_1 - \mathbb{E}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}\big]\Big)$$

$$-\frac{\lambda_1^\tau}{\gamma}\Big(\mathbb{E}\big[v_1 \mid \mathcal{F}_i^{(1),\tau}\big] - \Omega_{\mathrm{SD}}^\tau p_1 - (1-\Omega_{\mathrm{SD}}^\tau)\,\mathbb{E}\big[p_2 \mid \mathcal{F}_i^{(1),\tau}\big] - \gamma h_1^i\Big), \quad (64)$$

where these quantities are in the matching-session branch. The first term is the difference between the two effective prices, the exchange's Ω_1^τ and the matching session's $\Omega_{\mathrm{SD}}^\tau$, applied to the expected price change. The period 1 exchange demand does not appear because, on this branch, a unit bought in the exchange and a unit bought in the matching session are perfect substitutes.Both affect z_1^i one for one and both settle at p_1, and the matching session demand re-optimizes z_1^i whatever D_1^i was. Only the total period 1 trade is pinned down, by (48). What remains in (64) is the expected marginal value of shifting one unit from the matching session to the exchange, which is nonzero because the two routes carry different price impacts (λ_1^τ and $\lambda_{1,2}^\tau$ for the exchange, $\lambda_{\mathrm{SD},2}^\tau$ for the matching session). With a matching session for certain, the period 1 FOC therefore does not determine D_1^i. This is the degeneracy when $\theta = 1$ discussed in the main text.

Probabilistic matching session. The period 1 demand is submitted before it is known whether the matching session occurs. Let $\zeta = 1$ if it occurs and $\zeta = 0$ otherwise. The period 1 FOC weights the marginal expected utilities on the two branches by $1-\theta$ and θ. $(1-\theta)$ times κ_v^τ times the difference between the left- and right-hand sides of (61) on the no-matching-session branch, plus θ times the right-hand side of (64), equals zero. Since (64) does not involve D_1^i, the combined FOC has the same form as (61) on the no-matching-session branch, with combined coefficients:

$$\bar\kappa_v^\tau\,\mathbb{E}\big[v_1 - \gamma(h_1^i + D_1^i) \mid \mathcal{F}_i^{(1),\tau}\big] - \lambda_1^\tau D_1^i = (1-\theta)\,\kappa_v^\tau\Big(\Omega_1^\tau\, p_1 + (1-\Omega_1^\tau)\,\mathbb{E}\big[p_2 \mid \mathcal{F}_i^{(1),\tau}, \zeta = 0\big]\Big)$$
$$-\frac{\theta\,\lambda_1^\tau}{\gamma}\left(\Big(\Omega_{\mathrm{SD}}^\tau + \frac{\gamma\kappa_v^\tau}{\lambda_1^\tau}\big(\Omega_{\mathrm{SD}}^\tau - \Omega_1^\tau\big)\Big)\,p_1 + \Big(1 - \Omega_{\mathrm{SD}}^\tau - \frac{\gamma\kappa_v^\tau}{\lambda_1^\tau}\big(\Omega_{\mathrm{SD}}^\tau - \Omega_1^\tau\big)\Big)\,\mathbb{E}\big[p_2 \mid \mathcal{F}_i^{(1),\tau}, \zeta = 1\big]\right), \quad (65)$$

where

$$\bar\kappa_v^\tau := (1-\theta)\,\kappa_v^\tau - \theta\,\frac{\lambda_1^\tau}{\gamma}, \quad (66)$$

and in each term κ_v^τ and Ω_1^τ are evaluated on the same branch as the conditional expectation in that term. The no-matching-session branch contributes κ_v^τ to the expected marginal value and its effective price from (61). The matching-session branch contributes $-\lambda_1^\tau/\gamma$, because on that branch an extra unit of exchange demand only increases price impact, and its effective

price is (64) rearranged as a weighted combination of p_1 and the expected period 2 price. In both parentheses the weights sum to one.

The right-hand side of (65) is $\bar{\kappa}_v^\tau$ times a weighted average of p_1 and the two branch forecasts of p_2. Collecting the weight on p_1, and using $\kappa_v^\tau \Omega_1^\tau = 1$ on the no-matching-session branch,

$$\bar{\Omega}_1^\tau := \frac{1}{\bar{\kappa}_v^\tau}\left[(1-\theta) - \theta\Big(\kappa_v^\tau\big(\Omega_{\mathrm{SD}}^\tau - \Omega_1^\tau\big) + \frac{\lambda_1^\tau}{\gamma}\,\Omega_{\mathrm{SD}}^\tau\Big)\right], \tag{67}$$

with the matching-session branch quantities inside the parentheses. The remaining weight $1-\bar{\Omega}_1^\tau$ multiplies a forecast of p_2 that averages cross the two branches,

$$\begin{aligned}\bar{\kappa}_v^\tau\left(1-\bar{\Omega}_1^\tau\right)\bar{\mathbb{E}}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}\big] &:= (1-\theta)\,(\kappa_v^\tau - 1)\,\mathbb{E}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}, \zeta = 0\big] \\ &\quad + \theta\Big(\kappa_v^\tau\big(\Omega_{\mathrm{SD}}^\tau - \Omega_1^\tau\big) - \frac{\lambda_1^\tau}{\gamma}\big(1-\Omega_{\mathrm{SD}}^\tau\big)\Big)\mathbb{E}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}, \zeta = 1\big], \end{aligned} \tag{68}$$

where κ_v^τ in the first term is from the no-matching-session branch and the objects in the second term are from the matching-session branch. The two coefficients on the right sum to $\bar{\kappa}_v^\tau(1-\bar{\Omega}_1^\tau)$, so $\bar{\mathbb{E}}[p_2 \mid \mathcal{F}_i^{(1),\tau}]$ is a weighted average of the two branch's forecasts. Solving (65) for D_1^i:

$$\implies D_1^i(p_1) = \frac{\gamma\bar{\kappa}_v^\tau}{\gamma\bar{\kappa}_v^\tau + \lambda_1^\tau}\left(\frac{1}{\gamma}\Big(\mathbb{E}\big[v_1 \,\big|\, \mathcal{F}_i^{(1),\tau}\big] - \bar{\Omega}_1^\tau\, p_1 - \big(1-\bar{\Omega}_1^\tau\big)\,\bar{\mathbb{E}}\big[p_2 \,\big|\, \mathcal{F}_i^{(1),\tau}\big]\Big) - h_1^i\right). \tag{69}$$

This is the same form as (30), (48), and (63). There's a bid-shading weight, which is χ_1^τ with $\bar{\kappa}_v^\tau$ in place of κ_v^τ, times the competitive demand at an effective price that weights p_1 by $\bar{\Omega}_1^\tau$ and the forecast of p_2 by $1-\bar{\Omega}_1^\tau$. When $\theta = 0$, $\bar{\kappa}_v^\tau = \kappa_v^\tau$, $\bar{\Omega}_1^\tau = \Omega_1^\tau$, the forecast is the no-matching-session forecast, and (69) reduces to (63).

Matching coefficients. Conjecturing the linear period 1 demand schedule

$$D_1^i = a_{1,S}^\tau S_1^i + a_{1,h}^\tau h_1^i + a_{1,p}^\tau p_1, \tag{70}$$

substituting the projections (82)–(83) into (69), and matching coefficients on each direction $\xi \in \{S, h, p_1\}$,

$$a_{1,\xi}^\tau = \frac{\gamma\bar{\kappa}_v^\tau}{\gamma\bar{\kappa}_v^\tau + \lambda_1^\tau}\left(\frac{1}{\gamma}\Big(c_{\xi,1}^\tau - \bar{\Omega}_1^\tau\,\mathbf{1}\{\xi = p_1\} - \big(1-\bar{\Omega}_1^\tau\big)\,\bar{\rho}_{\xi,2}^\tau\Big) - \mathbf{1}\{\xi = h\}\right), \tag{71}$$

where $\bar{\rho}_{\xi,2}^\tau$ is the loading of $\bar{\mathbb{E}}[p_2 \mid \mathcal{F}_i^{(1),\tau}]$ on ξ, the average of the two branches' $\rho_{\xi,2}^\tau$ with the weights in (68). This has the same structure as (53) with $\bar{\Omega}_1^\tau$ in place of $\Omega_{\mathrm{SD}}^\tau$.

E. Cross-Period Demand Directions and Cross-Period Price Impacts

To evaluate the value of a deviation today, we need to account for the adjustment of future trades while remaining optimal. Thus, we differentiate the later FOCs with respect to the earlier decisions. Given present and future price impacts, the responsiveness of future demands to deviations of decisions today solves a system of equations.

E.1. Matching session → period 2 Impacts

The following cross-period impact is derived by differentiating the period 2 FOC (29) with respect to μ_i. Differentiating, we get

$$-\gamma\Big(\frac{N+M-1}{N+M}+\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}\Big)=\lambda^\tau_{\mathrm{SD},2}+\lambda^\tau_2\,\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}$$

$$\implies \frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}=-\frac{\gamma\frac{N+M-1}{N+M}+\lambda^\tau_{\mathrm{SD},2}}{\gamma+\lambda^\tau_2}. \tag{72}$$

E.2. period 1 exchange to period 2 impacts

A period 1 deviation also affects allocation through rivals' matching session demands, which respond to the deviation's effect on p_1. Since $z_1^i=\hat{z}_1^i+\mu_i-Q/(N+M)$,

$$\frac{\mathrm{d}z_1^i}{\mathrm{d}D_1^i}=1+\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i},\qquad \frac{\mathrm{d}D^i_{SD}}{\mathrm{d}D_1^i}=\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}, \tag{73}$$

where

$$\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}=\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}+(n_\tau-1)\,b_z^\tau\,a_{1,p}^\tau\,\lambda_1^\tau+n_{-\tau}\,b_z^{-\tau}\,a_{1,p}^{-\tau}\,\lambda_1^\tau. \tag{74}$$

Differentiating the period 2 FOC with respect to D_1^i:

$$0=-\gamma\Big(1+\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}+\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}\Big)-\lambda^\tau_{1,2}-\lambda^\tau_2\,\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i} \tag{75}$$

$$\implies \frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}=-\frac{\gamma\left(1+\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}\right)+\lambda^\tau_{1,2}}{\gamma+\lambda^\tau_2}. \tag{76}$$

E.3. period 1 exchange → SD Impacts

The period 1 exchange → SD impacts are given by differentiating the matching session FOC (45) with respect to the exchange decision at period 1:

$$\begin{aligned} 0 = & -(1-\delta)\,\gamma\Big(1-\frac{1}{N+M}\Big)\Big(1+\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}\Big)-\lambda_1^\tau\Big(1-\frac{1}{N+M}\Big) \\ & -\delta\,\gamma\Big(1-\frac{1}{N+M}+\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}\Big)\Big(1+\frac{\mathrm{d}\mu_i}{\mathrm{d}D_1^i}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}+\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}\Big) \\ & -\delta\,\lambda_{\mathrm{SD},2}^\tau\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}-\delta\,\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}\,\lambda_{1,2}^\tau. \end{aligned} \tag{77}$$

E.4. Own- and Cross-Period Price Impacts

Own-period price impacts need to satisfy

$$\lambda_t^\tau \equiv \frac{\mathrm{d}p_t}{\mathrm{d}D_t^i} = -\Big(\sum_{j\neq i}\frac{\partial D_t^j}{\partial p_t}\Big)^{-1} = \frac{1}{(n_\tau-1)\,b_t^\tau + n_{-\tau}\,b_t^{-\tau}}, \tag{78}$$

where b_2^τ is the period 2 demand slope of (34) and $b_1^\tau := |a_{1,p}^\tau| = -a_{1,p}^\tau$ is the period 1 demand slope on p_1 (in equilibrium $a_{1,p}^\tau < 0$).

Both cross-period price impacts are obtained by implicit differentiation of period 2 market clearing. For a period 1 deviation by trader i,

$$\lambda_{1,2}^\tau \equiv \frac{\mathrm{d}p_2}{\mathrm{d}D_1^i} = -\Big(\sum_{j\neq i}\frac{\partial D_2^j}{\partial p_2}\Big)^{-1}\Big(\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}+\sum_{j\neq i}\frac{\partial D_2^j}{\partial \hat{z}_2^j}\frac{\mathrm{d}\hat{z}_2^j}{\mathrm{d}D_1^i}+\sum_{j\neq i}\frac{\partial D_2^j}{\partial \mathcal{F}_j^{(2)}}\frac{\mathrm{d}\mathcal{F}_j^{(2)}}{\mathrm{d}D_1^i}\Big),$$

and analogously for a stage 1B deviation in the matching session demand μ_i. From the period 2 demand (30), a rival j of type τ' has $\frac{\partial D_2^j}{\partial p_2} = \frac{c_{p_2,2}^{\tau'}-1}{\gamma+\lambda_2^{\tau'}}$ and $\frac{\partial D_2^j}{\partial \hat{z}_2^j} = \frac{-\gamma}{\gamma+\lambda_2^{\tau'}}$, and responds to the public information in $\mathcal{F}_j^{(2)}$ that the deviation affects, p_1 and Q, with slopes $\frac{c_{p_1,2}^{\tau'}}{\gamma+\lambda_2^{\tau'}}$ and $\frac{c_{Q,2}^{\tau'}}{\gamma+\lambda_2^{\tau'}}$. The following is a common denominator:

$$\mathcal{D}_2^\tau \;:=\; \sum_{j\neq i}\frac{\partial D_2^j}{\partial p_2} \;=\; (n_\tau-1)\,\frac{c_{p_2,2}^{\tau}-1}{\gamma+\lambda_2^\tau} \;+\; n_{-\tau}\,\frac{c_{p_2,2}^{-\tau}-1}{\gamma+\lambda_2^{-\tau}}. \tag{79}$$

period 1 exchange →2 impact ($t=1$).

$$\lambda_{1,2}^\tau \;=\; -\frac{1}{\mathcal{D}_2^\tau}\left(\frac{\mathrm{d}D_2^i}{\mathrm{d}D_1^i}+(n_\tau-1)\,\frac{-\gamma}{\gamma+\lambda_2^\tau}\left(\frac{\partial D_1^\tau}{\partial p_1}\,\lambda_1^\tau + b_z^\tau a_{1,p}^\tau\lambda_1^\tau - \frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}\right)\right.$$

$$+n_{-\tau}\,\frac{-\gamma}{\gamma+\lambda_2^{-\tau}}\left(\frac{\partial D_1^{-\tau}}{\partial p_1}\,\lambda_1^{\tau}+b_z^{-\tau}a_{1,p}^{-\tau}\lambda_1^{\tau}-\frac{1}{N+M}\frac{\mathrm{d}Q}{\mathrm{d}D_1^i}\right)$$
$$+\frac{(n_\tau-1)\,c_{p_1,2}^{\tau}\lambda_1^{\tau}+(n_\tau-1)\,c_{Q,2}^{\tau}\,\mathrm{d}Q/\mathrm{d}D_1^i}{\gamma+\lambda_2^{\tau}}+\frac{n_{-\tau}\,c_{p_1,2}^{-\tau}\lambda_1^{\tau}+n_{-\tau}\,c_{Q,2}^{-\tau}\,\mathrm{d}Q/\mathrm{d}D_1^i}{\gamma+\lambda_2^{-\tau}}\Bigg), \quad (80)$$

where $\partial D_1^{\tau'}/\partial p_1$ denotes the period 1 demand slope of a type-τ' rival, which equals $a_{1,p}^{\tau'}$ under (70). At $t=1$, each rival j's period 2 projection onto Q uses (74).

Matching session → period 2 impact (stage 1B).

$$\lambda_{\mathrm{SD},2}^{\tau} \;=\; -\frac{1}{\mathcal{D}_2^{\tau}}\Bigg(\frac{\mathrm{d}D_2^i}{\mathrm{d}\mu_i}+(n_\tau-1)\,\frac{\gamma}{(N+M)(\gamma+\lambda_2^{\tau})}+n_{-\tau}\,\frac{\gamma}{(N+M)(\gamma+\lambda_2^{-\tau})}$$
$$+\frac{(n_\tau-1)\,c_{Q,2}^{\tau}}{\gamma+\lambda_2^{\tau}}+\frac{n_{-\tau}\,c_{Q,2}^{-\tau}}{\gamma+\lambda_2^{-\tau}}\Bigg). \quad (81)$$

For both cross-period impacts, the first term is ($\mathrm{d}D_2^i/\mathrm{d}D_1^i$ for $\lambda_{1,2}$, $\mathrm{d}D_2^i/\mathrm{d}\mu_i$ for $\lambda_{\mathrm{SD},2}$). The second and third terms capture rivals' exchange-response through (for $\lambda_{1,2}$) p_1 or (at stage 1B) the SD-inventory channel $\mathrm{d}\hat{z}_2^j/\mathrm{d}\mu_i=-1/(N+M)$. The final two fractions collect the $c_{p_1,2}$ (only for $\lambda_{1,2}$) and $c_{Q,2}$ terms summed over the $(n_\tau-1)$ same-type and $n_{-\tau}$ opposite-type counterparties.

F. Learning (Bayesian Projections)

All projections are linear-Gaussian and computed by the standard projection theorem $\mathbb{E}[X|Y]=\mathrm{Cov}(X,Y)\,\mathrm{Var}(Y)^{-1}\,Y$ (or its multivariate generalization), where the joint covariance matrices are derived from the equilibrium primitive loadings of all state variables. The projections are type-specific because traders of types I and U have different signal precisions $\sigma_{\varepsilon,I}^2$ vs. $\sigma_{\varepsilon,U}^2$.

F.1. period 1

The trader observes $\mathcal{F}_i^{(1),\tau}=\{S_1^i,h_1^i,p_1\}$. Conjecture that trader i of type τ has linear beliefs at period 1. Two projections are needed:

$$\mathbb{E}[v_1\mid\mathcal{F}_i^{(1),\tau}]=c_{S,1}^{\tau}\,S_1^i+c_{h,1}^{\tau}\,h_1^i+c_{p_1,1}^{\tau}\,p_1, \quad (82)$$
$$\mathbb{E}[p_2\mid\mathcal{F}_i^{(1),\tau}]=\rho_{S,2}^{\tau}\,S_1^i+\rho_{h,2}^{\tau}\,h_1^i+\rho_{p_1,2}^{\tau}\,p_1. \quad (83)$$

The v_1 projections are used in the period 1 FOC (61) and the matching FOC (71). The p_2 projections are used in (61).

F.2. Workup Stage 1B

At stage 1B, $\mathcal{F}_i^{(\mathrm{SD}),\tau} = \mathcal{F}_i^{(1),\tau}$, so projections coincide with the period 1 projections (82)–(83). The key new object is the rival-aggregate projection

$$\pi_\xi^\tau := \text{loading on } \xi \text{ of } \mathbb{E}\left[\frac{1}{N+M}\sum_{j\neq i}\mu_j^* \;\middle|\; \mathcal{F}_i^{(\mathrm{SD}),\tau}\right], \tag{84}$$

for $\xi \in \{S, h, p_1\}$. These rival projections appear in the matching equations (57)–(58) and represent the information that trader i extracts about average rival matching session demands from their information set.

F.3. period 2

At period 2 the trader's information set expands to

$$\mathcal{F}_i^{(2),\tau} = \mathcal{F}_i^{(\mathrm{SD}),\tau} \cup \{S_2^i, h_2^i, p_2, [Q]\}, \tag{85}$$

where Q is included in Design 2 only. The relevant projection is

$$\mathbb{E}[v_2 \mid \mathcal{F}_i^{(2),\tau}] = c_{S_1,2}^\tau S_1^i + c_{S_2,2}^\tau S_2^i + c_{h_1,2}^\tau h_1^i + c_{h_2,2}^\tau h_2^i + c_{p_1,2}^\tau p_1 + c_{p_2,2}^\tau p_2 + c_{Q,2}^\tau Q. \tag{86}$$

G. *Summary of demands and prices*

For trader i of type τ, the equilibrium strategies and inventories are

$$\begin{aligned}
D_1^i &= a_{1,S}^\tau S_1^i + a_{1,h}^\tau h_1^i + a_{1,p}^\tau p_1 \\
\mu_i &= b_z^\tau\, \hat{z}_1^i + b_S^\tau\, S_1^i + b_h^\tau\, h_1^i \\
D_{SD}^i &= \mu_i - Q/(N+M), \qquad Q = \sum_j \mu_j \\
D_2^i &= a_{2,S_1}^\tau S_1^i + a_{2,S_2}^\tau S_2^i + a_{2,h_1}^\tau h_1^i + a_{2,h_2}^\tau h_2^i + a_{2,p_1}^\tau p_1 + a_{2,p_2}^\tau p_2 + a_{2,Q}^\tau Q \\
\hat{z}_1^i &= h_1^i + D_1^i \\
z_1^i &= \hat{z}_1^i + D_{SD}^i \\
\hat{z}_2^i &= z_1^i + h_2^i \\
z_2^i &= \hat{z}_2^i + D_2^i
\end{aligned}$$

where the strategy coefficients satisfy the matching conditions (71), (56), and (36). Let τ_j denote the type of trader j. Market clearing, $\sum_j D_t^j = 0$, then gives the prices

$$p_1 = -\frac{1}{\sum_j a_{1,p}^{\tau_j}} \sum_j \left(a_{1,S}^{\tau_j} S_1^j + a_{1,h}^{\tau_j} h_1^j \right)$$

$$p_2 = -\frac{1}{\sum_j a_{2,p_2}^{\tau_j}} \sum_j \left(a_{2,S_1}^{\tau_j} S_1^j + a_{2,S_2}^{\tau_j} S_2^j + a_{2,h_1}^{\tau_j} h_1^j + a_{2,h_2}^{\tau_j} h_2^j + a_{2,p_1}^{\tau_j} p_1 + a_{2,Q}^{\tau_j} Q \right)$$

Substituting recursively, every equilibrium object (inventories, prices, matching session demands) is a linear function of the primitive shocks, which is what the projections of Appendix AF use.

H. Fixed-Point Problem

The equilibrium projections must be consistent with the joint Gaussian distribution induced by the equilibrium strategies. In our implementation, this consistency is achieved via the iterative fixed-point procedure.

(1) Start from an initial guess of the strategy coefficients $(a_1^\tau, a_2^\tau, b^\tau)$ and price impacts $(\lambda_1^\tau, \lambda_2^\tau, \lambda_{1,2}^\tau, \lambda_{\mathrm{SD},2}^\tau)$.
(2) Build the joint primitive distribution: each equilibrium object (inventories, prices, matching session demands) is a linear function of the primitive shocks; their loadings are computed from the current strategy coefficients.
(3) Compute the projections $c_{\xi,t}^\tau, \rho_{\xi,2}^\tau, \pi_\xi^\tau$ via the Gaussian projection theorem applied to the joint primitive distribution.
(4) update strategies via the matching conditions (36), (56), and (71); update λ's via the price-impact fixed points (32)–(33), the matching session cross-period equation (81), and the 3×3 joint solve combining (75), (80), and (77).
(5) Iterate until convergence.

At convergence, the strategy and price-impact updates are consistent with the projections they induce.